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MODELLING GLOBAL DEMOGRAPHIC CHANGE: RESULTS FOR JAPAN

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Modelling Global Demographic Change: Results for Japan

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Abstract

This paper explores the impact of demographic change in a series of increasing complex economics models. The models range from a simple two-country symmetric theoretical model to an empirically based 4-country MSG3 model, which represents the characteristics of Japan, United States, Rest of OECD and Rest of World. We first explore the properties of the two-country theoretical model with both a global and a single country stylized demographic transition. The results are similar to those found in the approach of Bryant (2004). We then explore the same shock in models that are made more complex by increasingly representing the empirical characteristics of the global economy. We find that although the basic insights from simplest theoretical models continue to hold, the quantitative results change significantly when we focus particularly on the demographic shock in a model representing the empirical characteristics of Japan.

In a final part of the paper, we use the complete empirical global model to explore the likely impacts on Japan of the demographic change already experienced from 1970 and examine the likely changes to be experienced out to 2040.

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1 Introduction

Many countries in the world economy are undergoing significant demographic change, or are projected to over coming decades. Table 1 illustrates the expected changes in old age dependency ratios of a number of countries between 2000 and 2050 as projected by the United Nations in the "2002 Revision of population projections". While these demographic shifts are substantial, none is more dramatic than those projected for Japan where the elderly dependency ratio rises from 37.4% in 2000 to a projected 93.7% by 2050. This substantial projected change in the demographic structure of Japan is likely to have a significant impact on the Japanese macro-economy. Yet there is a complex story behind the many facets of the change in demographic structures and how these might impact on an economy. In addition, Japan has already experienced several decades of demographic change and it is interesting to explore what impacts demographic change has already had on the Japanese economy.

This paper has a number of goals. It first sets out a methodology for capturing key aspects of the macroeconomic story in a global economic model. We focus on the impacts on labour supply, consumption and saving responses and then how in general equilibrium these responses impact on investment, trade and capital flows and asset markets. The basic approach extends the methodology of Blanchard (1985), Weil (1989), Faruqee, Laxton, and Symansky (1997) and Faruqee (2000a, 2000b, 2003a, 2003b) to modelling consumption and saving behaviour. The extension to allow for children follows Bryant et al (2001, 2002, 2004) and McKibbin and Nguyen (2002). This paper is part of a series of papers jointly researched with Ralph Bryant and his colleagues using the Multimod approach to modelling and a team at ANU working with the MSG3 approach to modelling. The Bryant series of papers has tended to focus on critical theoretical extensions to the basic approach and to explore key sensitivities of the theoretical approach, more recently focussing on pension systems and fiscal implications of demographic change. The approach of McKibbin and Nguyen has been to attempt to implement the theoretical breakthroughs into more realistic models of the global economy. The focus in particular is to develop an empirical model of Japan for understanding the quantitative as well as qualitative issues facing the Japanese macro-economy. This paper focuses on the demographic transition in Japan. The next stage of this project is to explore how the demographic transition being experienced in the rest of the world is likely to impact on the Japanese economy.

In Section 2, the recent experience and future projections of demographic change in Japan are summarized. In Section 3, a small stylized analytical model, similar to that in Bryant and McKibbin (2004) is developed. This is a very simple framework using approximations of multiple cohorts, based on a symmetric two country world where both countries are calibrated to the US economy. This simplified model is important for understanding the key analytical drivers. It is also consistent with the model developed by the Bryant team[1] which is based on the analytical approach of the Multimod model. Results for a stylized demographic shock are analyzed in section 4. In Section 5, starting with an almost common basis across the two modelling frameworks, we then add the empirical rigidities found in the MSG3 model (such as a mix of optimizing and backward looking households and firms) to the simple model and compare the theoretical representation to the MSG3 approach to the simple analytical model. Because of the symmetric nature of the model we are able to explore the impact of a stylized fertility decline that occurs in a single country (a large open economy) versus one that occurs globally (the world is a large closed economy). In section 6, we then extend this approach to a 4 country version of the MSG3 model in which the full macroeconomic and demographic representations of the four regions (United States, Japan, Rest of the OECD, and developing countries) are modelled. The real country asymmetries are captured in this more complete model. The model of Japan in this framework is the standard MSG3 model of Japan with 2 sectors of production (energy and non-Energy). This empirical representation of Japan is then simulated using the same stylized fertility decline as in the analytical model but in the context of the Japanese economy. This gives an insight into the impact of the stylized demographic change that is the same shock as used in Bryant (2004) for comparison purposes when moving from a simple theoretical representation of a generic economy to a model that is based on Japanese data. In Section 7, we derive and simulate the actual demographic change projected by the United Nations 2003 Population projections together with the historical experience of Japan to explore the likely contribution of the actual decline in Japanese fertility rates from 1970 to 2030. A conclusion and future directions for research are contained in Section 8.

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[1] Despite the attempt to line the models up as much as possible there are some real analytical differences between the models reflecting the different modeling approaches. There are also some key parameter differences discussed below.

2 Demographic Change in Japan

There is already a large and growing literature on the many aspects of demographic change in Japan[2] although few of these papers focus on the international aspects of Japan's demographic shock. Exceptions are the recent work by Faruqee (2000a, 2003b). In this paper we explore the issue of demographic change in Japan using a global modelling framework with a number of important decisions endogenous to the model, such as labour supply, human wealth accumulation, consumption and saving decisions, asset accumulation, investment demand and a full portfolio of asset prices.

The consequences of Japanese demographic change on Japan, however, is only part of the story, since global demographic change is likely to also impact on the Japanese economy in the coming decades. Future papers using the techniques applied in this paper will take a more global view, focussing on both the demographic shock in Japan and the demographic changes projected in the rest of the world.

Table 2 presents a more detailed breakdown of the demographic transition in Japan from 1950 to 2050. This is based on the 2002 Revision of the United Nations population projection (mid case) which contains substantial revisions compared to the earlier 1998 revision which was the basis of McKibbin and Nguyen (2002). It is clear from this table that the demographic adjustment in Japan is well under way.

A characteristic of the changing demographics in Japan is the decline in fertility with births per year (in thousands) falling from 2073 in 1970-75 to 1213 in the latest available data for 1995-2000. This is projected to decline to 940 by 2040-2050 and translates into a birth rate decline from 1.92% to 0.75% by 2015, although there is significant uncertainty about these types of projections[3]. Another characteristic of the Japanese demographic transition is the increased life expectancy from 63.9 years at birth in 1950 to 80.05 by 1995-2000 and this is further

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[2] See for example Endo and Katayama (1998), Horioka (1991), Meredith (1995), Ogawa and Retherford (1993), Takayama and Kitamura (1999), Takayama (1998), Takayama, Kitamura and Yoshida (1998), Yashiro and Oishi (1997), Yashiro (1998), Yashiro, Oshio, and Matsuya (1997).

[3] See Lee (2003) for a discussion.

projected rise to 87.9 by 2040-2050. Despite this increased longevity, the death rate per thousand is expected to rise from 956 in 1995-2000 to 1593 in 2040-50. This rise in the crude death rate is because the elderly are an increasing share of the total population. All of these factors work in the one direction of an increasingly aging Japanese population. In contrast to many other countries these demographic trends also imply a shrinking Japanese population beginning this decade.

3 A Theoretical Framework for Incorporating Demographic Change in a Multi-Country Model

There are two parts of the theoretical framework used in this paper. The first uses the MSG3 multi-country model and the second embeds in this general equilibrium model, a theoretical approach to modelling demographic change. In this section we will first summarize the MSG3 model and then present the demographic assumptions.

3.1 The MSG3 Model

The MSG3 multi-country model is based on the theoretical structure of the G-Cubed model outlined in McKibbin and Wilcoxen (1999)[4]. More details can be found in Appendix 1. A number of studies—summarized in McKibbin and Vines (2000)—show that the G-cubed model has been useful in assessing a range of issues across a number of countries since the mid-1980s.[5] Some of the principal features of the model are as follows:

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[4] Full details of the model including a list of equations and parameters can be found online at: www.gucubed.com

[5] These issues include: Reaganomics in the 1980s; German Unification in the early 1990s; fiscal consolidation in Europe in the mid-1990s; the formation of NAFTA; the Asian crisis; and the productivity boom in the US.

- The model is based on explicit *intertemporal* optimization by the agents (consumers and firms) in each economy[6]. In contrast to static CGE models, time and dynamics are of fundamental importance in the G-Cubed model.

- In order to track the macro time series, however, the behaviour of agents is modified to allow for short run deviations from optimal behaviour either due to myopia or to restrictions on the ability of households and firms to borrow at the risk free bond rate on government debt. For both households and firms, deviations from intertemporal optimizing behaviour take the form of rules of thumb, which are consistent with an optimizing agent that does not update predictions based on new information about future events. These rules of thumb are chosen to generate the same steady state behaviour as optimizing agents so that in the long run there is only a single intertemporal optimizing equilibrium of the model. In the short run, actual behaviour is assumed to be a weighted average of the optimizing and the rule of thumb assumptions. Thus aggregate consumption is a weighted average of consumption based on wealth (current asset valuation and expected future after tax labour income) and consumption based on current disposable income. Similarly, aggregate investment is a weighted average of investment based on Tobin's q (a market valuation of the expected future change in the marginal product of capital relative to the cost) and investment based on a backward looking version of Q.

- There is an explicit treatment of the holding of financial assets, including money. Money is introduced into the model through a restriction that households require money to purchase goods.

- The model also allows for short run nominal wage rigidity (by different degrees in different countries) and therefore allows for significant periods of unemployment depending on the labour market institutions in each country. This assumption, when taken together with the explicit role for money, is what gives the model its "macroeconomic" characteristics. (Here again the model's assumptions differ from the standard market clearing assumption in most CGE models.)

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[6] See Blanchard and Fischer (1989) and Obstfeld and Rogoff (1996).

- The model distinguishes between the stickiness of physical capital within sectors and within countries and the flexibility of financial capital, which immediately flows to where expected returns are highest. This important distinction leads to a critical difference between the *quantity of physical capital* that is available at any time to produce goods and services, and the *valuation of that capital* as a result of decisions about the allocation of financial capital.

As a result of this structure, the MSG3 model contains rich dynamic behaviour, driven on the one hand by asset accumulation and, on the other by wage adjustment to a neoclassical steady state. It embodies a wide range of assumptions about individual behaviour and empirical regularities in a general equilibrium framework. The interdependencies are solved out using a computer algorithm that solves for the rational expectations equilibrium of the global economy. It is important to stress that the term 'general equilibrium' is used to signify that as many interactions as possible are captured, not that all economies are in a full market clearing equilibrium at each point in time. Although it is assumed that market forces eventually drive the world economy to a neoclassical steady state growth equilibrium, unemployment does emerge for long periods due to wage stickiness, to an extent that differs between countries due to differences in labour market institutions.

3.2 A Theoretical Approach to Modelling Demographics

The theoretical framework used in this paper is based on that of Bryant and McKibbin (2001), applied to the MSG3 multi-country model which is summarized in Appendix 1. For the purposes of this paper, the MSG3 model has been extended to include demographic considerations, such that economic agents in the model now possess finite life-spans, and their incomes vary as they age. Specifically, economic agents progress from being financially dependent children to eventually being adults who are financially responsible for their own children. This section draws heavily on Faruqee (2000a, 2000b), who extended the Blanchard (1985) model of finitely-lived agents to include aging considerations. It is very similar to Bryant and Velculescu (2002) and Bryant (2004) in the way in which children are modelled. A key difference however is that in this paper we assume that all adults are assumed to bear the cost of providing support for children rather than having this support depend on the adult's age[7].

3.2.1 Adult Population

We begin by considering the adults in the population. In each period, a cohort of children matures and joins the adult population. The size of the newly matured cohort, at time *s*, with respect to the existing adult population, *N*(*s*) is referred to as the maturity rate, *b*(*s*). The maturity rate and its relationship to the population of children will be addressed in another section, below. Following Blanchard, we make the simplifying assumption that at any time *s*, all agents in the economy face the same mortality rate[8], *p*, defined here as the probability of any given agent dying before the next period. The number of adults who matured at a previous time *s*, who are still alive at a subsequent time *t* is given by:

$$n(s,t) = b(s)N(s)e^{-p(t-s)} \tag{1}$$

The adult population size can then be determined for any time *t* by summing the number of living adults from all of the cohorts that have ever matured:

$$\begin{aligned} N(t) &= \int_{-\infty}^{t} n(s,t)\; ds \\ &= \int_{-\infty}^{t} b(s)N(s)e^{-p(t-s)}\, ds \end{aligned} \tag{2}$$

where *N*(*t*) represents the adult population size, at time *t*.

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7 Bryant and Velculescu (2002) show the sensitivity of the results to this assumption. We are unable to implement this in the more complex model of Japan below and therefore use this assumption in the simple theoretical model for comparison purposes.

8. Blanchard notes that the assumption of a common mortality rate is a reasonable approximation for adults within the ages of 20 to 40. The fact that children and retirees, whose behaviour is of interest in studies of population aging, fall outside of this age bracket certainly indicates that the issue requires further attention.

Taking the derivative with respect to time yields an equation governing the evolution of the adult population size over time:

(3) $$\frac{\dot{N}(t)}{N(t)} = b(t) - p$$

The above equation has a simple interpretation: the adult population grows at a rate determined by the maturity rate less the mortality rate.

3.2.2 Child Population

In every period, a cohort of children is born. If we think of the adult population as representing the set of potential parents, then it follows that the size of a newly born cohort will depend upon the current adult population size and the birth rate, b_m. The expression for the number of children born at time *s* who are still alive at a later time *t*, is thus given by:

(4) $$m(s,t) = b_m(s)N(s)e^{-p(t-s)}$$

The aggregate number of children, $M(t)$, can be calculated by summing the number of surviving children, who were born recently enough that they have not yet reached adulthood. If we let Δ represent the fixed number of years from when a child is born to when it reaches adulthood, i.e. the period of childhood[9], then:

(5) $$M(t) = \int_{t-\Delta}^{t} m(s,t)\, ds$$

(6) $$M(t) = \int_{t-\Delta}^{t} b_m(s)\, N(s) e^{-(t-s)p}\ ds$$

Differentiating with respect to time:

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9. In the simulations that follow, the period of childhood is defined as the first 16 years of an agent's life; upon reaching his or her 16th birthday, the agent becomes classified as an adult.

(7) $$\dot{M}(t) = -pM(t) + b_m(t)N(t) - b_m(t-\Delta)N(t-\Delta)e^{-p\Delta}$$

(Note that in the final exponential, *pΔ* refers to the period of childhood multiplied by the mortality rate, it does not represent a change in *p*).

3.2.3 Relationship Between the Birth Rate and the Maturity Rate

Of the children who were born at time *t-Δ*, those who survive will mature at time *t*, at which time they are added to the adult population. Thus, the maturity rate at time *t* is dependent on the birth-rate, and adult population size, of *Δ* years past; as well as the mortality rate.

(8) $$b(t)N(t) = b_m(t-\Delta)N(t-\Delta)e^{-p\Delta}$$

Now, we know that:

(9) $$\begin{aligned} N(t-\Delta) &= N(t)e^{-\int_{t-\Delta}^{t} b(s)-p\, ds} \\ &= N(t)e^{p\Delta-\int_{t-\Delta}^{t} b(s)\, ds} \end{aligned}$$

so given the birth rate of *Δ* years ago, and the maturity rates over the last *Δ* years, we can determine the current maturity rate:

(10) $$b(t) = b_m(t-\Delta)e^{-\int_{t-\Delta}^{t} b(s)ds}$$

Since the maturity rates over the last *Δ* years will be dependent on previous values of the birth rate, we can see that the rate of maturity is predetermined by any given series of birth rates.

3.2.4 Adult Consumption

Adults attempt to maximise the expected utility derived from their lifetime consumption. Adults must take into account the uncertainty of their life-spans and thus they discount their planned future consumption by the probability that they may not survive through to future periods. Assuming a logarithmic utility function, each agent will maximise the following:

$$\max \int_t^{\infty} \ln c(s,v) e^{-(\theta+p)v}\, dv \tag{11}$$

subject to the budget constraint:

$$\dot{w}(s,t) = [r(t) + p] w(s,t) + y(s,t) - c(s,t) \tag{12}$$

where $c(s,t)$ is the consumption, at time t, of an adult who matured at time s, θ is the rate of time preference, $w(s,t)$ is the financial wealth that an adult who matured at time s holds at time t; and $r(t)$ is the interest rate earned on financial wealth. In addition to interest payments, adults also earn a rate of p on their holdings of financial wealth, due to the assumption of a life insurance market, as in Blanchard. Children do not play a part in the life insurance market, nor do they earn interest, as they are assumed to hold no financial wealth.

The optimal consumption path for an adult can be shown to be:

$$c(s,t) = (\theta + p)\left[w(s,t) + h(s,t)\right] \tag{13}$$

where $c(s,t)$ is the consumption, at time t, of an adult who matured at at time s, and $h(s,t)$ represents the human wealth of the adult. An adult's human wealth is defined as the present value of the adult's expected income over the remainder of his or her lifetime:

$$h(s,t) = \int_t^{\infty} e^{-\int_t^{v} r(i)+p\, di} y(s,v)\ \ dv \tag{14}$$

At any time t, then, the sum of financial wealth and human wealth—$w(s,t)$ and $h(s,t)$—represents an adult's total wealth: the means by which the agent can pay for his or her future consumption. Adults consume a proportion of their total wealth each period, the proportion being determined by their rate of time preference, and their likelihood of perishing before the next period.

Aggregate adult consumption, aggregate financial wealth and aggregate human wealth are simply the sums of the consumption, financial wealth and human wealth for all adults in the economy.

(15) $$C_N(t) = \int_{-\infty}^{t} c(s,t)n(s,t)\,ds$$

(16) $$W(t) = \int_{-\infty}^{t} w(s,t)n(s,t)\,ds$$

(17) $$H(t) = \int_{-\infty}^{t} h(s,t)n(s,t)\,ds$$

where $C_N(t)$ represents aggregate adult consumption, $W(t)$ is aggregate financial wealth, and $H(t)$ is aggregate human wealth.

The adult aggregate consumption function can be shown to be given by:

(18) $$C_N(t) = (\theta + p(t))\left[W(t) + H(t)\right]$$

3.2.5 Labour Supply, and Demographic Considerations

Empirically, one of the key economic characteristics that changes with age is the income that a person receives. We thus introduce age-earnings profiles into the model, such that an agent's income is determined by his or her age. Further, we assume that only adults earn labour income, and that children are dependent upon adults. Faruqee (2000a) utilises hump-shaped age-earnings profiles for adults, fitted to Japanese data using non-linear least squares (NLS). Intuitively, the hump-shaped profile of age-earnings reflects the fact that young adults generally have incomes that are increasing as the young individuals age and gain more experience. After a certain age, however, earnings decline, reflecting first the decreasing productivity associated with aging, and then eventually reflecting retirement behaviour.

Individual income is not specified as suddenly dropping to zero, at a given retirement age, for two reasons. Firstly, in practice, people typically retire at various ages, and some retirees continue to earn alternative forms of income even after retirement. Secondly, a discontinuous age-earnings profile introduces complications with respect to implementation in the MSG3 model.

We model the evolution of income over the lifecycle by beginning with the assumption that individuals are paid a wage for each unit of effective labour that they supply. We also assume that effective labour supply is a function of an individual's age and of the current state of technology. Aside from aging considerations, note that as time passes, the technological progress in the economy has a positive effect on the value of effective labour supplied by all agents.

The effective labour supply, at time *t*, of an agent who has been an adult since time *s*, is given by:

(19) $$l(s,t) = e^{ut}[a_1 e^{-\alpha_1(t-s)} + a_2 e^{-\alpha_2(t-s)} + (1 - a_1 - a_2)e^{-\alpha_3(t-s)}]\ ;$$ (a_i>0, α_i>0 for i=1 to 3)

The $e^{\mu t}$ component (where μ is the rate of technological progress) captures productivity increases due to advancements in technology. The remaining terms represent the non-linear functional form used to estimate the hump-shaped profile. The a_i and α_i parameters are estimated, based on empirical data, using NLS[10]. The hump-shaped effective labour supply specification will in turn lead to a hump shaped age-earnings profile.

Individual labour supply can be re-written as:

(20) $$l(s,t) = \sum_{i=1}^{3} l_i(s,t)$$

where:

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[10] Values used in this paper for Japan are as estimated by Faruqee for Japan: $\alpha_1 = 0.073$, : $\alpha_2 = 0.096$, : $\alpha_3 = 0.085$ and $a_1 = a_2 = 200$. In the theoretical model, to be consistent with Bryant (2004) we use the US parameters for both countries: $\alpha_1 = 0.08152$, : $\alpha_2 = 0.12083$, : $\alpha_3 = 0.10076$ and $a_1 = a_2 = 200$.

(21) $$l_i(s,t) = e^{\mu t} a_i e^{-\alpha_i (t-s)}; \qquad (a_i > 0, \alpha_i > 0)$$

and:

(22) $$a_3 = (1 - a_1 - a_2)$$

Thus, the evolution of an adult's labour supply over time is given by:

(23) $$\dot{l}(s,t) = \sum_{i=1}^{3} (\mu - \alpha_i) l_i(s,t)$$

Aggregate effective labour supply in the economy for any time *t*, *L*(*t*), is the sum of the effective labour supplied by all adults in the economy:

(24) $$\begin{aligned} L(t) &= \int_{-\infty}^{t} n(s,t) l(s,t)\, ds \\ &= \sum_{i=1}^{3} L_i(t) \end{aligned}$$

where:

(25) $$L_i(t) = \int_{-\infty}^{t} n(s,t) l_i(s,t)\, ds$$

It can then be shown that:

(26) $$\begin{aligned} \dot{L}(t) &= L_1(t) + L_2(t) + L_3(t) \\ &= (\mu - \alpha_1 - p) L_1(t) + (\mu - \alpha_2 - p) L_2(t) + (\mu - \alpha_{3i} - p) L_3(t) + e^{\mu t} b(t) N(t) \end{aligned}$$

The intuition behind the equation above is that the aggregate labour supply of the economy changes as the entire population ages, and also as new agents mature into the labour force.

Figure 1 shows the Alternative approximations for the age earnings profile for the United States. Figure 2 shows the data for the Age Earnings profiles in Japan from 1970 to 1997

3.2.6 Intergenerational Transfer

In our stylised model, children differ from adults, in that they do not provide labour supply (and thus do not receive payment for labour) and they do not hold financial wealth. Children are dependent upon their parents; each child receives an intergenerational transfer every period, $c(t)$, which is completely consumed by the child. As they do not make any consumption decision, but rather just entirely consume their transfer, we do not need to account for their human wealth.

We assume that $c(t)$ grows at the rate of productivity growth, μ—as the economy becomes more efficient in production, children benefit.

$$c(t) = c_0 e^{\mu t} \tag{27}$$

The simplest specification[11] for adult transfer payments is to assume that adults share the burden of supporting children equally, i.e.

$$j(s,t) = j(t) \tag{28}$$

where $j(s,t)$ is the payment that an individual adult, who became an adult at time s, is liable for at time t. Note that transfer payments are bound by the following budget constraint, which constrains aggregate child receipts to equal aggregate adult payments:

$$c(t)M(t) = \int_{-\infty}^{t} j(t)n(s,t)\,ds \tag{29}$$

Thus:

$$j(t) = \frac{c(t)M(t)}{\int_{-\infty}^{t} n(s,t)\,ds} \tag{30}$$

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11. Bryant and Velculescu (2001) for example make most expenses for children fall on younger adults whereas we assume that adults of all ages contribute equally.

(31) $$j(t) = c(t)\delta(t)$$

Aggregate consumption for the whole economy, then, is the sum of aggregate adult consumption and aggregate child consumption:

$$C(t) = (\theta + p)[A(t) + H(t)] + c(t)M(t) \qquad (1)$$

3.2.7 Income and Human Wealth

Previously, individual human wealth was defined as the expected present-value of future income over an adult's remaining lifetime. Having defined the profile of labour supply over the lifecycle, we can now be more explicit with respect to income. An adult's income is after-tax labour income, plus government transfers, less lump sum taxes and intergenerational transfers:

(32) $$y(s,t) = [1 - \tau(t)]w(t)l(s,t) + tr(t) - tx(t) - j(t)$$

where $y(s,t)$ denotes the income, at time t, of an adult who matured at time s; $l(s,t)$ is the individual effective labour supply; $\tau(t)$ is the marginal tax rate; and $w(t)$ is the wage paid per unit of effective labour. We assume that the distribution of lump sum taxes, tx, and government transfers, tr, is uniform across the population, thus the year of an individual's coming of age is not a determinant of either of these two variables.

We define aggregate adult income as:

(33) $$Y(t) = \int_{-\infty}^{t} y(s,t)n(s,t)\, ds$$

Taking the time derivative of $h(s,t)$, after substituting in the expression for individual income, we obtain:

(34) $$\dot{h}(s,t) = [r(t) + p]h(s,t) - [1 - \tau(t)]w(t)l(s,t) - [tr(t) - tx(t) - j(t)]$$

The intuition for the equation above is that as time passes, future earnings are no longer as distant in time, and should therefore be discounted by a lesser magnitude—this explains the (r + p) growth—while at the same time, some income has just been received, and thus can no longer be considered part of human wealth—this explains why the current period's income is subtracted.

We can show that the evolution of aggregate human wealth is governed by the following relationship:

$$\dot{H}(t) = r(t)H(t) - Y(t) + h(t,t)n(t,t) \quad (35)$$

The intuition behind the equation above is that aggregate human wealth changes over time as future income draws nearer, thus H grows at the rate of r; the presence of death, and hence p, does not affect *aggregate* human wealth, because insurance companies redistribute the wealth of the dead. Further, in each period, people receive income, and having been received, it can no longer be considered human wealth. The last term on the right hand side represents the new human wealth that the newly-matured cohort brings to the economy, each period.

4 Results from a theoretical two-region model

This section uses a stylized two-region model based on the theoretical specification of the MSG3 model with some key simplifications. We assume first that there are no backward looking agents[12]. Thus consumption and investment is assumed to be undertaken by fully optimizing agents. Indeed if the probability of death was zero, this model would be a fully Ricardian model in which the rate of time preference determines the real rate of interest at each point of time. The introduction of a probability of death (as in Blanchard (1985)) implies a finite lifetime for all agents and the pure Ricardian equivalence propositions no longer apply.

In the calibration here, we follow Bryant (2004) and choose the birth rate of children of 2.50408% per year and an infant mortality rate of 0.075%. This generates an adult maturity rate

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[12] Bryant (2004) also assumes a proportion of backward looking consumers in his theoretical model.

of 2% per year. We also assume that the adult mortality rate is 0.15% and productivity growth in the economies is 2% per year.

To solve the model, we first normalize all quantity variables by each economy's endowment of effective labour units. In the case of some variables such as population we normalize by per capita rather than effective units. This means that in the steady state all real variables are constant in these units although the actual levels of the variables will be growing at the underlying rate of growth of population plus productivity. Next, we must make base-case assumptions about the future path of the model's exogenous variables in each region. In all regions we assume that the long run real interest rate is 5 percent, tax rates are held at their 2002 levels and that fiscal spending is allocated according to 2002 shares.

A crucial group of exogenous variables are productivity growth rates by sector and country. The usual baseline assumption in the MSG3 and G-Cubed models is that the pattern of technical change at the sector level is similar to the historical record for the United States (where data is available). Both regions in the theoretical model of this section are identical.

Given these assumptions, we solve for the model's perfect-foresight equilibrium growth path over the period 2002-2200. This a formidable task: the endogenous variables in *each* of the 199 periods need to be solved out and include, among other things: the equilibrium prices and quantities of each good in each region, intermediate demands for each commodity by each industry in each region, asset prices by region and sector, regional interest rates, bilateral exchange rates, incomes, investment rates and capital stocks by industry and region, international flows of goods and assets, labour demanded in each industry in each region, wage rates, current and capital account balances, final demands by consumers in all regions, and government deficits.[13] At the solution, the budget constraints for all agents are satisfied, including both intra-temporal and inter-temporal constraints.

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[13] Since the model is solved for a perfect-foresight equilibrium over a 199 year period, the numerical complexity of the problem is on the order of 80 times what the single-period set of variables would suggest. We use software summarized in McKibbin and Sachs (1991) Appendix C, for solving large models with rational expectations on a personal computer.

In this section we introduce a stylized fall in fertility rates to the theoretical model. This follows the same shock as in Bryant (2004)[14]. This is shown in Figure 3. The birth rate of children begins to decline from year 1 and reaches a trough of 1.15 percentage points by year 53. This fall in the birth rate of children impacts on the disposable income of adults immediately (through lower transfers to children) but does not impact on the maturity rate of adults until year 19. The maturity rate (or adult birth rate) reaches a trough of -0.81556 percent points by 71 years into the simulation. In Bryant (2004) both countries experience a demographic transition, however, one country undergoes a faster transition. In the current paper only one country experiences a demographic transition. To enable comparison with Bryant (2004) we have scaled the transition in the country to be the size of the **difference** between transitions in Bryant (2004). We also ultimately return the birth rates to the original levels after a long period rather than permanently changing birth rates. Returning the birth rate in the very long run to the original level has to be done for technical reasons related to the numerical solution technique. However, the results for the first century of the shock is unaffected by this difference in long run assumptions. The key is that the relative shocks occurring in both countries are the same in the two studies.

We consider two alternatives regarding this demographic shock. The first is that the shock occurs simultaneously in both countries. This is equivalent to a closed economy because neither the exchange rate nor trade and capital flows will change. We then assume that the shock occurs in one country only, in order to get an insight into the likely impacts of the demographic shock on bilateral trade and asset flows and real exchange rate adjustment.

In the following figures, all results are expressed as deviation from the baseline solution of the model, either in percentage, percentage points or however indicated on the figures.

Results for both countries are contained in Figures 4 through 7. Following Bryant (2004) and earlier papers we label one country "US" for the United States and the other country

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[14] Note that in the implementation in the 4 country MSG3 model we assume children are up to 16 years of age due to data requirements. In the theoretical model we adopt the Bryant convention of a child being anyone up to age 18. The difference this makes to the results are small.

"ZZ". The "ZZ" country is the country experiencing the asymmetric demographic shock. This could be considered a stylized representation of Japan except that the parameters are based on US data in this simple model. These figures show the deviation from the baseline for the global shock (labelled as "closed economy") which is the same for each country. The figures also show the results for both countries when the shock only occurs in the ZZ economy.

4.1 Symmetric Shock

When the shock is the same in both countries, the assumption of model symmetry shows that there is no change in exchange rates or trade and current account balances. These results illustrate what would happen in a "large closed economy". The shock is a gradual decline in the birth rate. Thus the disposable incomes of households (after deduction for supporting children) effectively rise in the first 18 years as there are fewer children to support. The real economic impacts on labor supply occurs when there are less children maturing into adults and entering the work force 19 years after the initial shock. Recall that effective labour inputs are calculated using age earnings profiles so that as the cohort of lower birth rate adults move through the workforce, the effective loss of workers is magnified by the loss in workers when they move through their more productive years. The decline in labor supply has the biggest per unit impact at around age 40, or 40 years after the demographic shock began because this is the most productive stage of the "missing workers". The demographic transition lasts well past 100 years by which time the initial shocks are returned to zero.

As expected with a significant fall in number of workers, the aggregate macroeconomic variables began to show sharp decline about 40 years after the initial shock. Real GDP is 60% lower after 100 years compared to the baseline with no demographic transition. Aggregate consumption adjusts more quickly as individual households attempt to smooth their consumption over their lifetime. The story at the individual level is quite different to the aggregate story. In the short run households individually attempt to smooth their consumption given expected future changes in individual income and expected future changes in aggregate variables such as real interest rates that affect their intertemporal decisions. Households initially cut their consumption slightly using the fact that there is less need for spending on children to spread this across future consumption and also in response to lower relative price of future consumption (i.e. lower real interest rates). Per capita GDP also rises since there is a realization that future output will need more capital per worker to sustain production, which stimulates investment. Figure 4 shows that the capital output ratio in sector 2 (non energy) rises significantly for the first 8 decades. The rise in investment stimulates the economy and raises per capita GDP for 5 decades.

An important aspect of the shock is the effect on real interest rates. Real interest rates fall over time to be 0.9 percentage points (90 basis points) less than otherwise. The global nature of the shock means that there is now an imbalance between saving and investment as households attempt to push their current consumption into the future. There is no other country to cushion the shock with. Investment rises but by less than savings and thus real interest rates fall to bring savings and investment back into line. Another way to interpret this result is that with fewer workers over time the marginal product of capital must fall. This is reflected through the real arbitrage in the model between the marginal product of capital and the real interest rate which drives real interest rates lower until the capital stock can adjust.

In terms of quantitative outcomes, the result for real interest rates is a key difference to Bryant (2004). The reason for this difference is the assumption in the MSG3 model that the intertemporal elasticity of substitution is unity (i.e. log utility) where Bryant (2004) assumes an intertemporal elasticity of substitution of 0.5[15]. As a direct result of this assumption, the change in interest rates in Bryant (2004) is more than double the change in interest rates in figure 6. The intertemporal elasticity of substitution is a critical parameter in both studies and is the subject of a wide ranging debate in the literature.

4.2 Asymmetric Shock

Results for the shock that only occurs in the "ZZ" country are also shown in figures 4 through 6. It is clear that in the country experiencing the shock, results are very similar to those for the global shock. The key lesson is that the availability of other countries not experiencing

[15] See Appendix 2 page 47 of Bryant et al (2003) for a clear exposition of the role and empirical importance of the value of the intertemporal elasticity of substitution of consumption.

the demographic shock enables some capacity through the balance of payments, to reduce the impact of the shock in the "ZZ" country. The aggregate effects are less for both GDP and consumption. The rise in savings as a result of the action of individual households in the ZZ country now translates into a current account surplus for many decades at the beginning of the shock. This saving finds a higher rate of return outside the ZZ country. This shows up initially as a current account surplus and a trade surplus. Over time however the return on the savings are repatriated back to the ZZ economy and this shows up as a swing into trade balance deficit. Note that one country's deficit must be another's surplus.

The ability of households and firms in the ZZ country to use the US to help smooth the shock implies that there is less of a need for capital deepening in the ZZ country. Equity market values rise and fall by less (as shown by Tobin's q in figure 5). This smoothing is shown even more clearly in the outcome for consumption per adult and GDP per adult in figure 5. As expected the real interest rate outcomes are also less extreme because changes in the current account allow saving and investment to be different.

One interesting aspect of the adjustment is the path of the real exchange rate shown in Figure 6. The outflow of capital in the short run as households and firms intertemporally adjust causes a depreciation of the real exchange rate for the US. Over time the real exchange rate appreciates as this capital is repatriated to the ZZ country to finance future consumption. On top of this asset allocation effect, there is also a real fundamental adjustment in the real exchange rate. The utility function of households in both countries consists of a CES function of all goods. Thus the fact that there are fewer goods produced by the ZZ country (both energy and non-energy) available over time as production rises less quickly, implies that the relative price of these goods will rise. Thus a key driver of the long term real exchange rate outcome is the appreciation of the real exchange rate of the ZZ country (or the rise in the relative price of its production). This effect is quite large as would be expected given the shift in relative economic weight of the two countries. After a century the real exchange rate appreciates by more than 40 percent. This is the same story as in Bryant (2004) although the magnitudes are different for the same reason as for the interest rate outcomes. There is less adjustment of asset prices when the intertemporal elasticity of substitution is higher, as in this paper.

Given the path of the real exchange rate, the nominal exchange rate path is driven by the assumption about monetary policy. In this paper, in contrast to Bryant (2004) and earlier papers using this model, we now assume that monetary authorities follow a modified Henderson McKibbin Rule[16] in which they adjust the nominal short term interest rates based on the lagged nominal interest rate and the gap between actual and desired inflation and the gap between actual and potential growth rates of real output. Clearly from Figure 6, with an appreciation of the real exchange rate of country ZZ and eventual depreciation of the nominal exchange rate of country ZZ, the price level of country ZZ must rise relative to the US price levels. Monetary authorities in ZZ are slow to offset the effects of rising inflation due to a decline in the growth of capacity output. The monetary policy assumptions have some impact in the very short run but largely impact on the outcomes of nominal variables and price levels over time.

Another aspect of the adjustment through the balance of payments is the accumulation of foreign assets (figure 6) which is the counterpart of the current account surpluses. Within the trade balance the share of imports in GDP rises more for the ZZ country for the asymmetric shock than for the global shock as agents attempt to adjust their relative consumption baskets. As a result of the demographic shock in the ZZ country, there are relatively less ZZ goods and relative more US goods available.

This stylized model has many useful insights that parallel those in Bryant (2004) although there are differences due to different parameterizations of the models. It is interesting to add further complexity to this framework to see how robust the basic insights are. This is done in the following sections.

5 Simplest Model versus a stylized MSG3 Model

In Section 4 we used the simplest benchmark theoretical model that is consistent with the approach of the MSG3 model to explore the analytical story behind demographics shocks.

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[16] See Levin et al (1999) and Appendix 1.

Ultimately we are also interested in the quantitative magnitudes of the shocks. This requires a model that more completely represents the actual characteristics of economies. As a step in moving from the simplest world to a more complex world, it is useful to take an intermediate step to introduce some rigidity in agent's behaviour that is found in the larger MSG3 model, while maintaining the 2 country symmetry of the model. In this section, following the standard approach in the MSG3 model, we assume that 70% of households do not re-optimize continually but follow an optimal rule of thumb where their consumption is a proportion of current income (both from labour and returns on financial assets). Similarly we assume that 70% of firms are using an error correction model for Tobin's Q in which the Q determining their investment gradually adjusts to the true underlying Tobin's Q.

We continue to maintain the assumption that countries are symmetric and of equal size. The question of country size and structure will be shown to matter once we move to more realistic models of Japan in the next section.

The results for this Symmetric MSG3 model are shown in figures 7 through 9. The main result is that the movement away from complete intertemporal optimization does not change the qualitative story very much within the countries but does change the interactions between countries. The main difference from introducing less forward looking behaviour is that there is less smoothing of consumption (as would be expected). This shows in Figure 7 where there is a smaller current account surplus for the first two decades. This contrasts which much larger swings in the current account deficits in later years. A similar difference can be found in the adjustment of the trade balance between the two countries. The lack of smoothing through the balance of payments also shows up in a larger fall in consumption after a century in the MSG3 model compared to the theoretical model.

Overall it seems that reducing the extent of optimization by agents seems to have significant quantitative but not qualitative impacts on the basic story about adjustments to demographic shocks. The overall story is dominated by the large demographic swings and how these feed into change in labour supply and output on the supply side and consumption and investment behaviour on the demand side. Perhaps the apparently small impact of reducing the extent of forward looking behaviour is not surprising, given that the introduction of finite lifetimes already reduces the forward looking-ness of households in the model.

6 Results in the 4 Region MSG3 Model

In this section we expand the theoretical world of two symmetric countries used in sections 4 and 5, to try and capture the actual characteristics of the world economy such as asymmetry in size and structure of countries. In this section we incorporate the same theoretical demographic assumptions used in the smaller models above, into the MSG3 model aggregated so that there are four main regions: Japan, The United States, the rest of the OECD (ROECD) and the rest of the world (ROW). As well as incorporating the key differences between these economies as captured in the structure of the full MSG3 model, we also change the demographic characteristics to suit each of the countries actual demographic features. This involves using actual estimated age earnings profiles for the United States, Japan and the rest of the OECD. For the ROW region we use the ROECD estimates.

The projected future paths of productivity growth also differ across countries. We continue to use the assumption for the United States from the 2 country model. In regions other than the United States, the sector-level rates of technical change are scaled up or down in order to match the region's observed average rate of aggregate productivity growth over the past 5 years. This approach attempts to capture the fact that the rate of technical change varies considerably across industries while reconciling it with regional differences in overall growth. It is important to note that we are not starting in a steady state which most intertemporal models do (such as Bryant (2004)) but we assume that the model is on the stable transition path towards a steady state. The point on this path is the actual 2002 data set that we calibrate the model to replicate.

We also start with exactly the same demographic shock as in the 2 country models above so as to have a benchmark for comparison. The current results will illustrate how different our insights might be when we allow for a number of key differences such as asymmetries in country production structures, consumption baskets, composition of trade flows, net asset positions etc. However we do not at this stage calculate individually what each difference makes but only what the entire group of differences make as a whole for the stylized shock.

Results are contained in figures 10 through 12 for a demographic shock that only occurs in Japan. The shock is assumed to occur in 2002 so that we can compare to the same shock in the 2 country models. In this case the results for Japan can be compared to the ZZ country in the 2 country models. Although we solve the model for 150 years, we truncate the results to focus on the first 80 years of the shock given the numerical problems of solving large model over several hundred years. It is important when comparing the results in figures 10 through 12 with figure 7 through 9 to remember that results for the 2 country model are presented over a much longer time period that the 4 country model.

Figure 10 shows that the aggregate results for GDP and consumption follow a similar profile to those for the 2 country symmetric MSG3 model, although the initial rise in GDP is more pronounced and the subsequent fall in GDP is larger for Japan in the 4 country model than for the generic ZZ country in the smaller models. The demographic shock is the same but the age earnings profiles in Japan are quite different to those in the small model (where we use age earnings profiles for the US in both the US and ZZ models to preserve symmetry). When the shock is passed through the actual age earnings profile for Japan, there is a larger effective shock. This results in a large fall in effective workers in Japan relative to the 2 country symmetric model which was based on the US age earnings profile. There is also a large rise in the current account surplus for Japan for the 8 decades shown which peaks at 1.5% of GDP after 25 years. This outcome reflects a range of issues but in particular appears sensitive to the larger initial holding of net foreign assets in Japan compared to the zero holdings in the 2 country symmetric models.

It is interesting that although the Japan in the 4 country model is a much smaller share of the global economy than the ZZ country is in the small models (i.e. ZZ is 50% of the world), the effects on asset markets of the Japanese demographic shock within Japan are still large. Real interest rates in Japan fall by over 2 percentage points after 80 years compared to 0.5 percentage points for ZZ in the 2 country models. The appreciation of the nominal exchange rate is also twice as large in Japan model after 80 years (60% versus 25%) compared to that for ZZ.

Introducing a more realistic representation of the Japanese economy, including it's relative economic size, trade structure, initial asset balances, structure of production and consumption and Japanese age earnings profiles quantitatively change the results we found in the simple two country stylized model. However, the qualitative story from the 2 country models remains robust to the more realistic representation of the Japanese economy. This is an important result because it means that the theoretical advances in other research such as Bryant (2004) appear to give useful insights for policymakers on the likely qualitative story behind a large global demographic transition. It is perhaps not surprising that although the basic story from the smaller symmetric models remains robust, the quantitative magnitudes of effects are sensitive to the quantitative specification of the model.

7 The Impact of Demographic Change in Japan Since 1970

The earlier sections examined the impact of a stylized demographic shock within increasingly complex models of the world economy. In the last section we compared a stylized demographic shock in Japan with the small models. In this section we present results for a demographic shock in Japan that corresponds to the actual changes since 1970 and projected changes from 2000. The goal is to see what contribution demographic change might have made to the Japanese macroeconomic experience since 1970 and what might be expected over coming decades.

There is an interesting methodological issue about simulating demographic shocks in a model with rational expectations. In the four country model we have assumed that children become adults after 16 years. Thus a surprise change in the birth rate today will be a perfectly predicted change in the adult population 17 years hence in a model with rational agents. In most papers that ignore children, changes in birth rates are assumed to be the arrival of new adults into the work force. The simulation we undertake is a realisation that there is a change in the birth rate of children from 1970 onwards (based on UN actual birth rates and projected rates past 2000). This implies in 1970 there is also a known shock to the adult population from 1986 onwards. In comparing this simulation with the results from our earlier study without children, it should be kept in mind that in the model with children, we are assuming that in 1970, agents in the model expect the adult birth rate to change from 1986 onwards and have a good idea of the macroeconomic implications of this. For a strict comparison to the earlier results we could assume that the change in the adult birth rate occurs in 1986 by complete surprise, even though the change in the adult birth rate was the result of a change in the birth of children from 1970 onwards. This latter assumption corresponds to the simulation in McKibbin and Nguyen (2001) in which there were no children. Although we remain uneasy about the assumptions behind the simulations presented here and how much it is in practice reasonable to assume that people understood there was a demographic transition in 19970 to impact on 1986, these results are illustrative of the likely consequences for Japan of the demographic change currently under way.

In simulating the model from 1970, we re-bench the demographic model of age structure etc to be at the 1970 structure and change the net asset positions of countries but we keep the other data calibration as for the standard MSG3 model. Thus this is not a complete recalibration of the entire model based on 1970 but a partial recalibration of key initial conditions. We first solve the model from 1970 to 2100 in order to get a baseline in which no further demographic shocks are present. We then commence the counterfactual simulation in 1970 on the assumption that the demographic shock becomes news in that year. As discussed above, the assumption that the demographic shock is unanticipated until 1970 might be regarded as problematic for a number of reasons. In a model with rational expectations we have little choice than to make that assumption. Given we are interested in what is likely to happen from the current year for the next several decades, this assumption may not be such a problem for the analysis. However, the reader is cautioned to interpret the results from 1970 to 2000 with great care. Nonetheless they give some insight into the likely impact of the demographic transition already occurring in Japan.

A second important qualification is that we are not imposing a demographic transition in the other countries outside Japan in these results. This assumption unambiguously has important implications. What we are calculating is the impact of changes in Japanese demographic structures rather than being more comprehensive in replicating all of the historical shocks. The results are not forecasts but the contribution of demographic change to the historical outcomes.

Table 3 contains projections from the United Nations Population Division World Population Prospects: The 2002 Revision (Medium Variant Projections). These are converted into adult population growth rates (defined as the growth in the adult population) and the maturity rate, which is the rate at which children become adults as a proportion of the adult population. Think of this as the adult birth rate (in terms of our earlier discussion). It is clear that to fit the actual experience into the assumption of our modelling framework requires some simplifying assumptions. In the actual data, the mortality rate varies over time as does the mortality rate by cohort. We have to convert these numbers into rates consistent with the assumption of a constant mortality rate over time and over cohorts. The key variable that is tied down is the adult population growth rate so that we capture the changes in the labour force over time. The child birth rate is then calculated to give this aggregate outcome.

Figure 13 shows the maturity rate and adult population growth rate in the UN projections out to 2050. Figure 14 shows the birth rate, maturity rate and adult mortality rate over the entire period. This figure contains three lines. The mortality rate is the death rate for adults. We assume that the death rate for children is 0.75%. The birth rate is the birth rate of children (expressed a percent of the adult population) and the maturity rate is the rate at which children become adults in terms of the adult population (simply thought of as the "adult birth rate"). To get the child birth rate, we take the maturity rate from the UN projections and our assumption about the mortality rate and back out the implied birth rate of children in terms of the adult population. None of the figures contain the more conventional notation of a population growth rate since we express everything in terms of adults not totals for reasons outlined in section 3.

Another feature of Figure 14 is that it reflects the UN projections out to 2050 and then an arbitrary assumption that the rates return to the baseline rates gradually after 2050. This assumption will clearly matter for results past 2050 and for the expectations of earlier periods about the future. For our purposes it is numerically simpler to assume reversion. Some experimentation suggests that this assumption has little impact on the results from 1970 to 2020, which is our main focus. However clearly this matters for later years. In summary this shock can be characterized as a sharp fall in the birth rate of children from 1970 followed from 1986 by a fall in the maturity rate (i.e. rate of emergence of new adults into the working adult population). After 2050, the birth rate gradually returns to baseline. This is similar in many ways to the stylized shock used in sections 4 to 6 of this paper and the initial decades of the demographic shocks in Bryant (2004).

Figure 15 shows the impact of the assumptions on the child population, the adult working population and the effective labour supply, shown as percent deviation from what would have been the case in the baseline, without the change in the birth rate. The overshooting of effective labour supply, relative to the population decline, reflects the aging of the population and the movement of the current working population along the age earnings profile with declining effectiveness as they move past the peak earning years of 40-45 year of age. The profile of the effective labour force (relative to the baseline) seems rather large but reflects the size of the estimated coefficients of the age earnings profile from Japanese data (based on Faruqee (2000a)). Future work will explore the sensitivity of the results to these parameters.

Figures 16 through 18 shows results for Japan for the shock summarized above. These results are expressed as deviation from the baseline (which has the estimated age earnings profiles but no changes in birth rates) expressed as either percent deviation from baseline or percentage point deviation from baseline as indicated. It is important to note that the underlying baseline without the demographic transition has underlying growth in Japan of close to 3% per year. Thus when we present results of variables falling relative to base this does not mean that they are falling in absolute terms. The slope of a curve showing percent deviation from baseline for variables is the deviation from baseline of the growth rate of that variable. Thus if GDP is falling relative to baseline by 1% per year it is still rising in absolute terms by 2% per year rather than 3% per year.

Figure 17 and 18 contains results for real GDP per adult and real consumption per adult as well as results for the broader macro economy. These results can also be interpreted as the relative contribution of the demographic adjustment in Japan to the changes in Japanese variables over time.

The realization that the demographic shock is to occur leads to a rise in private saving or a fall in consumption (fig 17, top right hand chart). When the adult population begins to fall from 1987, per capita saving begins to rise. At the same time aggregate saving falls because the population of adults is falling. An interesting aspect of the shock is that although real GDP growth is expected to eventually decline over time, there is a realization that the capital output ratio will need to rise in Japan as workers become scarce in the future. The initial rise in GDP is caused by a desire to gradually increase the capital stock through greater investment and through the improvement in the current account through a rise in net exports as Japanese saving channels overseas. The rise in investment and net exports causes a short term Keynesian style overshooting in GDP in the early years of the shock. This lasts for a number of decades until the large fall in the number of people in Japan leads to a fall in the capital stock in Japan (past 2050). The current account moves into surplus (relative to baseline) (figure 16) and stays in surplus until 2020. The trade balance gradually moves towards deficit as foreign investments made during the earlier period are repatriated from overseas.

The movement in the real exchange rate is consistent with the relative changes in saving and investment. In the long run, the consumption side of the model in which all households in all countries consume a share of Japanese goods in their consumption bundle implies that as less Japanese goods are produced, their relative value will rise. Thus as shown in the theoretical model, a long run real appreciation of the Yen is expected. In the short run, this effect partly determines the real exchange rate. The real exchange rate is also determined by the allocation of international financial capital. In the short run the saving response dominates the investment response and there is a depreciation of the yen in real terms (relative to the baseline which has an appreciating yen). The strong investment response during 2000 and onwards begins a period of appreciation relative to baseline. In earlier papers without children in the model, the investment response dominated the short run and a net capital inflow was found. However in that earlier paper the change in the labour force occurred in the first period and new capital was urgently needed. In this more realistically simulation the change in the labour force is expected to occur 17 years into the future thus the saving response dominates the investment response in the short run.

8 Summary and Conclusion

This paper has explored the implications of changing demographic structures in Japan by starting with a simple theoretical framework and moving to a complete global model with Japan a central focus. The starting point of the paper is aligned with the theoretical work in Bryant (2004) although we both branch out into different directions. This paper demonstrates that the insights in the various papers by the Brookings team using a theoretical framework based on the Multimod model gives broadly similar insights into the key issues and key sensitivity of results as the MSG3 approach. While Bryant et al (2003) and Bryant (2004) extend the theoretical approach that can subsequently be implemented in the MSG3 model; we have tried to undertake the extremely difficult agenda of implementing the theoretical developments into empirically based models of Japan.

The results in this paper suggest some important implications of current and expected demographic adjustment in Japan. The results may be specifically driven by key assumptions in the modelling framework used and substantially more research is required to understand key sensitivities. In particular we have found that many of the differences between the earlier results from the MSG3 approach and the approach in Bryant's stylized model can be traced to core assumptions about initial conditions or key parameters such as the intertemporal elasticity of substitution of consumption.

The paper shows that the type of demographic shock already underway in Japan and which is expected to persist for many decades has important impacts on the Japanese economy although it is very important to distinguish between the aggregate impacts versus the per capita impacts. According to the results in this paper the big aggregate adjustments in Japan are likely to occur commencing from around 2010. These include a fall in consumption and GDP growth **relative to baseline** but strong investment out to at least 2040 as capital is substituted for labour. Note that the baseline assumes GDP growth without any demographic transition of close to 3% per year so the actual growth rate in Japan with still be over 1% per year in absolute terms – the results show the deviation relative to that growth rate. Between 2010 and 2020 the slope of the deviation from base curve for GDP in figure 16 suggest a fall in the growth rate of GDP of 1% per year and 2% per year from 2020 relative to base. Also there is predicted to be a period of appreciating real exchange rates and falling real interest rates. To the extent that the investment response we currently observe in Japan may not appear consistent with these results can be interpreted in a number of ways. The demographic adjustment may be swamped by other shocks such as the problem of the current Japanese economic difficulties. The results may also point to a deficiency in the model. There needs to be an assessment of the range of shocks facing Japan during the 1990s, independently of the demographic shock, before a firm conclusion can be drawn[17]. It is also clearly important to include the demographic adjustments in the rest of the world before a firm conclusion can be drawn.

One key difference between the results in this paper and our earlier research summarized in McKibbin and Nguyen (2001) is the pattern of current account adjustment. In the earlier paper the current account moved into deficit initially. In the current paper there is a sustained period of current account surpluses accompanied by eventual deficits in the balance of trade. The key

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[17] See for example Callen and McKibbin (2001).

reason for this difference is in the nature of the demographic shock. In the earlier result we assumed that the falling adult population was a surprise when it started and thus there needed to be a rise in current investment, which was larger than the rise in current saving. In this paper the shock to the future adult population is anticipated well in advance once the realization of a changing child birth rate is observed. Thus the short-run response of saving begins immediately while the investment response is delayed until the additional capital required to replace the disappearing workers is needed. We also assume a higher productivity growth rate in this new model and hence current account changes can be sustained by productivity growth. The key to the balancing of the intertemporal budget constraint can be found in the adjustment of the trade balance (i.e. there can be permanent changes in the stocks of assets). Thus the current account moves into surplus in this paper rather than into deficit as was found in that earlier paper.

We have attempted to begin the analysis in this paper in line with the basic theoretical starting point of Bryant (2004) and have found many similarities in the results across the two different frameworks. Once we move away from the simple model we find results change just as they do once the issue of pension systems is introduced as in Bryant (2004). One of the issues which appear to have an important affect on the results is that the consequences of initial conditions matter for the subsequent adjustment. This was true in moving from the MSG3 model with 4 countries for the stylized shock and attempting to replicate the Japanese conditions since 1970. Some of the issues that emerge in creating the initial demographic data is outlined in Appendix 1 but there are a range of further sensitivity analysis required to fully understand key results. Another important issue is the fact that the full MSG3 model departs from the stylised model of Bryant and McKibbin (2001) by allowing imperfection in the model such as the degree of optimizing versus backward looking consumers and firms. The more inertia in the MSG3 model compared to the simple analytical models of that earlier work, the more dampened is the degree of intertemporal smoothing. Yet an equally important consideration is the intertemporal elasticity of substitution in consumption already demonstrated by Bryant et al (2003) to be important.

The results in this paper are preliminary and significantly more work is needed on modelling the nature of the demographic shock as well as sensitivity analysis on the key determinants of the macroeconomic outcomes. However over the past years of research we have been able to refine the modelling and understand better the key issues. Also Japan is not undergoing a demographic transition in isolation from the rest of the world and once the foreign demographic changes are incorporated into the analysis the results for Japan could be quite different. Nonetheless, the results to date suggest that the approach taken in this paper using the MSG3 model and the research in the Brookings Institution Project using the Multimod approach but focussing on theoretical developments is very promising and has generated important and robust insights. However as with most research the results of this paper raise more questions for future research.

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Appendix I: A Stylized MSG3 multicountry model

The model used in this paper is a reduced version of the MSG3 multi-country model. We aggregate the world into four regions: Japan; the United States; rest of the OECD and rest of the world. In the model used in Bryant and McKibbin (2001) the countries in the simplified MSG3 model were assumed to be symmetric representations of the US economy with simplified functional forms and full intertemporal optimising agents. In this paper we use the empirically based MSG3 model with the Japanese economy based on the Japanese economy in the MSG3 model and with rigidities in consumption and investment decisions found in the larger model.

In the remainder of this section we present the essence of the MSG3 model. The MSG3 model is based on the G-Cubed model of McKibbin and Wilcoxen (1999). The key difference between the MSG3 model and the G-Cubed model is that we aggregate the 12 sectors in each country to 2 sectors (energy and non-energy). This is very similar to the structure of the MSG2 model of McKibbin and Sachs (1991) except that we use the econometric estimation of the G-Cubed model to parameterise the model. The reader is referred to chapters 2 and 5 of McKibbin and Wilcoxen (2002) for greater detail on the analytical basis of the model.

The MSG3 model captures the behaviour of several economic agents: households, the government, the financial sector and 2 firms, one each in the 2 production sectors in each economy. The two sectors of production are energy and non-energy (this is much like the aggregate structure of the MSG2 model). The following gives an overview of the theoretical structure of the model by describing the decisions facing these agents in one of these countries. Throughout the discussion all quantity variables will be normalized by the economy's endowment of effective labour units. Thus, the model's long run steady state will represent an economy in a balanced-growth equilibrium.

A. Firms

We assume that each of the two sectors can be represented by a price-taking firm that chooses variable inputs and its level of investment in order to maximize its stock market value.

Each firm's production technology is represented by a constant elasticity of substitution (CES) function. Output is a function of capital, labor, energy and materials:

(1) $$Q_i = A_i^o \left(\sum_{j=k,l,e,m} \left(\delta_{ij}^o\right)^{1/\sigma_i^o} x_{ij}^{(\sigma_i^o - 1)/\sigma_i^o} \right)^{\sigma_i^o/(\sigma_i^o - 1)}$$

where Q_i is the output of industry i, x_{ij} is industry i's use of input j, and A_i^o, δ_{ij}^o, and σ_i^o are parameters. A_i^o reflects the level of technology, σ_i^o is the elasticity of substitution, and the δ_{ij}^o parameters reflect the weights of different inputs in production; the superscript o indicates that the parameters apply to the top, or "output", tier. Without loss of generality, we constrain the δ_{ij}^o's to sum to one.

The goods and services purchased by firms are, in turn, aggregates of imported and domestic commodities, which are taken to be imperfect substitutes. We assume that all agents in the economy have identical preferences over foreign and domestic varieties of each commodity. We represent these preferences by defining composite commodities that are produced from imported and domestic goods. Each of these commodities, Y_i, is a CES function of inputs domestic output, Q_i, and an aggregate of goods imported from all of the country's trading partners, M_i:

(2) $$y_i = A_i^{fd} \left(\left(\delta_{id}^{fd}\right)^{1/\sigma_i^{fd}} Q_i^{(\sigma_i^{fd} - 1)/\sigma_i^{fd}} + \left(\delta_{if}^{fd}\right)^{1/\sigma_i^{fd}} M_i^{(\sigma_i^{fd} - 1)/\sigma_i^{fd}} \right)^{\sigma_i^{fd}/(\sigma_i^{fd} - 1)}$$

where σ_i^{fd} is the elasticity of substitution between domestic and foreign goods.[18] For example, the energy product purchased by agents in the model are a composite of imported and domestic

[18] This approach follows Armington (1969).

energy. The aggregate imported good, M_i, is itself a CES composite of imports from individual countries, M_{ic}, where c is an index indicating the country of origin:

$$(3) \qquad M_i = A_i^{ff} \left(\sum_{c=1}^{7} \left(\delta_{ic}^{ff}\right)^{1/\sigma_i^{ff}} M_{ic}^{(\sigma_i^{ff}-1)/\sigma_i^{fd}} \right)^{\sigma_i^{ff}/(\sigma_i^{ff}-1)}$$

The elasticity of substitution between imports from different countries is σ_i^{ff}.

By constraining all agents in the model to have the same preferences over the origin of goods we require that, for example, the agricultural and service sectors have the identical preferences over domestic oil and imported oil.[19] This accords with the input-output data we use and allows a very convenient nesting of production, investment and consumption decisions.

In each sector the capital stock changes according to the rate of fixed capital formation (J_i) and the rate of geometric depreciation (δ_i):

$$(4) \qquad \dot{k}_i = J_i - \delta_i k_i$$

Following the cost of adjustment models of Lucas (1967), Treadway (1969) and Uzawa (1969) we assume that the investment process is subject to rising marginal costs of installation. To formalize this we adopt Uzawa's approach by assuming that in order to install J units of capital a firm must buy a larger quantity, I, that depends on its rate of investment (J/k):

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[19] This does not require that both sectors purchase the same amount of oil, or even that they purchase oil at all; only that they both feel the same way about the origins of oil they buy.

(5) $$I_i = \left(1 + \frac{\phi_i}{2}\frac{J_i}{k_i}\right) J_i$$

where ϕ_i is a non-negative parameter. The difference between J and I may be interpreted various ways; we will view it as installation services provided by the capital-goods vendor. Differences in the sector-specificity of capital in different industries will lead to differences in the value of ϕ_i.

The goal of each firm is to choose its investment and inputs of labour, materials and energy to maximize intertemporal net-of-tax profits. For analytical tractability, we assume that this problem is deterministic (equivalently, the firm could be assumed to believe its estimates of future variables with subjective certainty). Thus, the firm will maximize:[20]

(6) $$\int_t^{\infty} (\pi_i - (1-\tau_4)\, p^I I_i)\, e^{-(R(s)-n)(s-t)}\, ds$$

where all variables are implicitly subscripted by time. The firm's profits, π, are given by:

(7) $$\pi_i = (1-\tau_2)(p_i^* Q_i - w_i x_{il} - p_i^e x_{ie} - p_i^m x_{im})$$

where τ_2 is the corporate income tax, τ_4 is an investment tax credit, and p^* is the producer price of the firm's output. $R(s)$ is the long-term interest rate between periods t and s:

(8) $$R(s) = \frac{1}{st}\int_t^s r(v)dv$$

[20] The rate of growth of the economy's endowment of effective labor units, n, appears in the discount factor because the quantity and value variables in the model have been scaled by the number of effective labor units. These variables must be multiplied by exp(nt) to convert them back to their original form.

Because all real variables are normalized by the economy's endowment of effective labour units, profits are discounted adjusting for the rate of growth of population plus productivity growth, *n*. Solving the top tier optimization problem gives the following equations characterizing the firm's behavior:

(9) $$x_{ij} = \delta_{ij}^{o} \left(A_i^o\right)^{\sigma_i^o - 1} Q_i \left(\frac{p_i^*}{p_j}\right)^{\sigma_i^o} \quad j \in \{l, e, m\}$$

(10) $$\lambda_i = (1 + \phi_i \frac{J_i}{k_i})(1 - \tau_4)\, p^I$$

(11) $$\frac{d\lambda_i}{ds} = (r + \delta_i)\,\lambda_i - (1 - \tau_2) p_i^* \frac{dQ_i}{dk_i} - (1 - \tau_4) p^I \frac{\phi_i}{2}\left(\frac{J_i}{k_i}\right)^2$$

where λ_i is the shadow value of an additional unit of investment in industry *i*.

Equation (9) gives the firm's factor demands for labour, energy and materials and equations (10) and (11) describe the optimal evolution of the capital stock. Integrating (11) along the optimum trajectory of investment and capital accumulation, $(\hat{J}(t), \hat{k}(t))$, gives the following expression for λ_i:

(12) $$\lambda_i(t) = \int_t^{\infty} \left((1 - \tau_2) p_i^* \left.\frac{dQ_i}{dk_i}\right|_{\hat{J},\hat{k}} + (1 - \tau_4) p^I \frac{\phi_i}{2}\left(\frac{\hat{J}_i}{\hat{k}_i}\right)^2 \right) e^{-(R(s)+\delta)(s-t)} ds$$

Thus, λ_i is equal to the present value of the after-tax marginal product of capital in production (the first term in the integral) plus the savings in subsequent adjustment costs it generates. It is related to *q*, the after-tax marginal version of Tobin's Q (Abel, 1979), as follows:

(13)
$$q_i = \frac{\lambda_i}{(1-\tau_4)\, p^I}$$

Thus we can rewrite (10) as:

(14)
$$\frac{J_i}{k_i} = \frac{1}{\phi_i}(q_i - 1)$$

Inserting this into (5) gives total purchases of new capital goods:

(15)
$$I_i = \frac{1}{2\phi_i}\left(q_i^2 - 1\right)k_i$$

Based on Hayashi (1979), who showed that actual investment seems to be party driven by cash flows, we modify (15) by writing I_i as a function not only of q, but also of the firm's current cash flow at time t, π_i, adjusted for the investment tax credit:

(16)
$$I_i = \alpha_2 \frac{1}{2\phi_i}\left(q_i^2 - 1\right)k_i + (1-\alpha_2)\frac{\pi_i}{(1-\tau_4)p^I}$$

This improves the model’s ability to mimic historical data and is consistent with the existence of firms that are unable to borrow and therefore invest purely out of retained earnings.

So far we have described the demand for investment goods by each sector. Investment goods are supplied, in turn, by a third industry that combines labour and the outputs of other industries to produce raw capital goods. We assume that this firm faces an optimization problem identical to those of the other two industries: it has a nested CES production function, uses inputs of capital, labour, energy and materials in the top tier, incurs adjustment costs when changing its capital stock, and earns zero profits. The key difference between it and the other sectors is that we use the investment column of the input-output table to estimate its production parameters.

B. Households

We first describe the behaviour of households in the MSG3 model. How this is modified to incorporate demographic change in outlined in the following section. Households have three distinct activities in the model: they supply labour, they save, and they consume goods and services. Within each region we assume household behaviour can be modelled by a representative agent with an intertemporal utility function of the form:

(17) $$U_t = \int_t^{\infty} (\ln c(s) + \ln g(s))\, e^{-\theta(s-t)}\, ds$$

where $c(s)$ is the household's aggregate consumption of goods and services at time s, $g(s)$ is government consumption at s, which we take to be a measure of public goods provided, and θ is the rate of time preference.[21] The household maximizes (17) subject to the constraint that the present value of consumption be equal to the sum of human wealth, H, and initial financial assets, F:[22]

(18) $$\int_t^{\infty} p^c(s)c(s)e^{-(R(s)-n)(s-t)} = H_t + F_t$$

Human wealth is defined as the expected present value of the future stream of after-tax labour income plus transfers:

(19) $$H_t = \int_t^{\infty} (1 - \tau_1)(W(L^G + L^C + L^I + \sum_{i=1}^{12} L^i) + TR)\, e^{-(R(s)-n)(s-t)}\, ds$$

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[21] This specification imposes the restriction that household decisions on the allocations of expenditure among different goods at different points in time be separable.

[22] As before, n appears in (18) because the model's scaled variables must be converted back to their original basis.

where τ_1 is the tax rate on labour income, *TR* is the level of government transfers, L^C is the quantity of labour used directly in final consumption, L^I is labour used in producing the investment good, L^G is government employment, and L^i is employment in sector *i*. Financial wealth is the sum of real money balances, *MON*/*P*, real government bonds in the hand of the public, *B*, net holding of claims against foreign residents, *A*, the value of capital in each sector:

$$F = \frac{MON}{p} + B + A + q^I k^I + q^c k^c + \sum_{i=1}^{12} q^i k^i \tag{20}$$

Solving this maximization problem gives the familiar result that aggregate consumption spending is equal to a constant proportion of private wealth, where private wealth is defined as financial wealth plus human wealth:

$$p^c c = \theta(F + H) \tag{21}$$

However, based on the evidence cited by Campbell and Mankiw (1990) and Hayashi (1982) we assume some consumers are liquidity-constrained and consume a fixed fraction γ of their after-tax income (*INC*).[23] Denoting the share of consumers who are not constrained and choose consumption in accordance with (21) by α_8, total consumption expenditure is given by:

$$p^c c = \alpha_8 \theta(F_t + H_t) + (1 - \alpha_8)\gamma INC \tag{22}$$

The share of households consuming a fixed fraction of their income could also be interpreted as permanent income behaviour in which household expectations about income are myopic.

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[23] There has been considerable debate about the empirical validity of the permanent income hypothesis. In addition the work of Campbell , Mankiw and Hayashi, other key papers include Hall (1978), and Flavin (1981). One side effect of this specification is that it prevents us from computing equivalent variation. Since the behavior of some of the households is inconsistent with (21), either because the households are at corner solutions or for some other reason, aggregate behavior is inconsistent with the expenditure function derived from our utility function.

Once the level of overall consumption has been determined, spending is allocated among goods and services according to a CES utility function.[24] The demand equations for capital, labour, energy and materials can be shown to be:

(23) $$p_i x_i^c = \delta_i^c y \left(\frac{p^c}{p_i} \right)^{\sigma_c^o - 1}, i \in \{k, l, e, m\}$$

where y is total expenditure, x_i^c is household demand for good i, σ_c^o is the top-tier elasticity of substitution and the δ_i^c are the input-specific parameters of the utility function. The price index for consumption, p^c, is given by:

(24) $$p^c = \left(\sum_{j=k,l,e,m} \delta_j^c p_j^{\sigma_c^o - 1} \right)^{\frac{1}{\sigma_c^o - 1}}$$

Household capital services consist of the service flows of consumer durables plus residential housing. The supply of household capital services is determined by consumers themselves who invest in household capital, k^c, in order to generate a desired flow of capital services, c^k, according to the following production function:

(25) $$c^k = \alpha k^c$$

where α is a constant. Accumulation of household capital is subject to the condition:

(26) $$\dot{k}^c = J^c - \delta^c k^c$$

24 The use of the CES function has the undesirable effect of imposing unitary income elasticities, a restriction usually rejected by data. An alternative would be to replace this specification with one derived from the linear

We assume that changing the household capital stock is subject to adjustment costs so household spending on investment, I^c, is related to J^c by:

(27)
$$I^c = \left(1 + \frac{\phi^c}{2}\frac{J^c}{k^c}\right)J^c$$

Thus the household's investment decision is to choose I^C to maximize:

(28)
$$\int_t^\infty (p^{ck}\alpha k^c - p^I I^c)e^{-(R(s)-n)(s-t)}ds$$

where p^{ck} is the imputed rental price of household capital. This problem is nearly identical to the investment problem faced by firms and the results are very similar. The only important differences are that no variable factors are used in producing household capital services and there is no investment tax credit for household capital. Given these differences, the marginal value of a unit of household capital, λ_C, can be shown to be:

(29)
$$\lambda_c(t) = \int_t^\infty \left(p^{ck}\alpha + p^I \frac{\phi_c}{2}\left(\frac{\hat{J}_c}{\hat{k}_c}\right)^2 \right) e^{-(R(s)+\delta)(s-t)}ds$$

where the integration is done along the optimal path of investment and capital accumulation, $(\hat{J}_c(t), \hat{k}_c(t))$. Marginal q is:

(30)
$$q_c = \frac{\lambda_c}{p^I}$$

and investment is given by:

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expenditure system.

(31)
$$\frac{J_c}{k_c} = \frac{1}{\phi_c}\left(q_c - 1\right)$$

The Labour Market

We assume that labour is perfectly mobile among sectors within each region but is immobile between regions. Thus, wages will be equal across sectors within each region, but will generally not be equal between regions. In the long run, labour supply is completely inelastic and is determined by the exogenous rate of population growth. Long run wages adjust to move each region to full employment. In the short run, however, nominal wages are assumed to adjust slowly according to an overlapping contracts model where wages are set based on current and expected inflation and on labour demand relative to labour supply. The equation below shows how wages in the next period depend on current wages; the current, lagged and expected values of the consumer price level; and the ratio of current employment to full employment:

(32)
$$w_{t+1} = w_t \left(\frac{p_{t+1}^c}{p_t^c}\right)^{\alpha_5} \left(\frac{p_t^c}{p_{t-1}^c}\right)^{1-\alpha_5} \left(\frac{L_t}{\overline{L}}\right)^{\alpha_6}$$

The weight that wage contracts attach to expected changes in the price level is α_5 while the weight assigned to departures from full employment ($\overline{L}$) is α_6. Equation (32) can lead to short-run unemployment if unexpected shocks cause the real wage to be too high to clear the labour market. At the same time, employment can temporarily exceed its long run level if unexpected events cause the real wage to be below its long run equilibrium.

The Government

We take each region's real government spending on goods and services to be exogenous and assume that it is allocated among inputs in fixed proportions, which we set to 1996 values. Total government outlays include purchases of goods and services plus interest payments on government debt, investment tax credits and transfers to households. Government revenue comes from sales taxes, corporate and personal income taxes, and from sales of new government bonds. In addition, there can be taxes on externalities such as carbon dioxide emissions. The government budget constraint may be written in terms of the accumulation of public debt as follows:

$$\dot{B}_t = D_t = r_t B_t + G_t + TR_t - T_t \tag{33}$$

where B is the stock of debt, D is the budget deficit, G is total government spending on goods and services, TR is transfer payments to households, and T is total tax revenue net of any investment tax credit.

We assume that agents will not hold government bonds unless they expect the bonds to be paid off eventually and accordingly impose the following transversality condition:

$$\lim_{s \to \infty} B(s)\, e^{-(R(s)-n)s} = 0 \tag{34}$$

This prevents per capita government debt from growing faster than the interest rate forever. If the government is fully leveraged at all times, (34) allows (33) to be integrated to give:

$$B_t = \int_t^{\infty} (T - G - TR)\, e^{-(R(s)-n)(s-t)} ds \tag{35}$$

Thus, the current level of debt will always be exactly equal to the present value of future budget surpluses.[25]

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[25] Strictly speaking, public debt must be less than or equal to the present value of future budget surpluses. For tractability we assume that the government is initially fully leveraged so that this constraint holds with equality.

The implication of (35) is that a government running a budget deficit today must run an appropriate budget surplus as some point in the future. Otherwise, the government would be unable to pay interest on the debt and agents would not be willing to hold it. To ensure that (35) holds at all points in time we assume that the government levies a lump sum tax in each period equal to the value of interest payments on the outstanding debt.[26] In effect, therefore, any increase in government debt is financed by consols, and future taxes are raised enough to accommodate the increased interest costs. Other fiscal closure rules are possible, such as requiring the ratio of government debt to GDP to be unchanged in the long run. These closures have interesting implications but are beyond the scope of this paper.

Financial Markets and the Balance of Payments

The eight regions in the model are linked by flows of goods and assets. Flows of goods are determined by the import demands described above. These demands can be summarized in a set of bilateral trade matrices which give the flows of each good between exporting and importing countries.

Trade imbalances are financed by flows of assets between countries. Each region with a current account deficit will have a matching capital account surplus, and vice versa.[27] We assume asset markets are perfectly integrated across regions.[28] With free mobility of capital,

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[26] In the model the tax is actually levied on the difference between interest payments on the debt and what interest payments would have been if the debt had remained at its base case level. The remainder, interest payments on the base case debt, is financed by ordinary taxes.

[27] Global net flows of private capital are constrained to be zero at all times – the total of all funds borrowed exactly equals the total funds lent. As a theoretical matter this may seem obvious, but it is often violated in international financial data.

[28] The mobility of international capital is a subject of considerable debate; see Gordon and Bovenberg (1994) or Feldstein and Horioka (1980).

expected returns on loans denominated in the currencies of the various regions must be equalized period to period according to a set of interest arbitrage relations of the following form:

(36) $$i_k + \mu_k = i_j + \mu_j + \frac{\dot{E}_k^j}{E_k^j}$$

where i_k and i_j are the interest rates in countries k and j, μ_k and μ_j are exogenous risk premiums demanded by investors (calibrated in the baseline to make the model condition hold exactly with actual data), and E_k^j is the exchange rate between the currencies of the two countries.

Capital flows may take the form of portfolio investment or direct investment but we assume these are perfectly substitutable *ex ante*, adjusting to the expected rates of return across economies and across sectors. Within each economy, the expected returns to each type of asset are equated by arbitrage, taking into account the costs of adjusting physical capital stock and allowing for exogenous risk premiums. However, because physical capital is costly to adjust, any inflow of financial capital that is invested in physical capital will also be costly to shift once it is in place. This means that unexpected events can cause windfall gains and losses to owners of physical capital and *ex post* returns can vary substantially across countries and sectors. For example, if a shock lowers profits in a particular industry, the physical capital stock in the sector will initially be unchanged but its financial value will drop immediately.

Money Demand

Finally, we assume that money enters the model via a constraint on transactions.[29] We use a money demand function in which the demand for real money balances is a function of the value of aggregate output and short-term nominal interest rates:

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[29] Unlike other components of the model we simply assume this rather than deriving it from optimizing behavior. Money demand can be derived from optimization under various assumptions: money gives direct utility; it is a

(37) $$MON = PY\,i^{\varepsilon}$$

where Y is aggregate output, P is a price index for Y, i is the interest rate, and ε is the interest elasticity of money demand.

The supply of money is endogenously determined given a simple policy rule followed in each country. The generic form of the rule is a modified Henderson, McKibbin, Taylor Rule in which the short term nominal interest rate [30] depends on the lagged nominal interest rate; a weight on the gap between actual and desired inflation; a weight on the gap between actual and potential growth. In the more detailed model for some countries we also add a weight on the gap between the actual and desired values of the bilateral nominal exchange rate with the United States.

Assessing the Model

All models have strengths and weaknesses and the MSG3 model is no exception. Its most important strength is that it distinguishes between financial and physical capital and includes a fully integrated treatment of intertemporal optimization by households, firms and international portfolio holders. This allows the model to do a rigorous job of determining where physical capital ends up, both across industries and across countries, and of determining who owns the physical capital and in what currency it is valued. Overall, the key feature of the MSG3 model is its treatment of capital, and that is also what most distinguishes it from other models in either the macro, trade or CGE literatures.

The MSG3 model also has other strengths. All budget constraints are satisfied at all times, including both static and intertemporal budget constraints on households, governments and countries. Short-run behaviour captures the effects of slow wage adjustment and liquidity constraints, while long-run behaviour is consistent with full optimization and rational

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factor of production; or it must be used to conduct transactions. The distinctions are unimportant for our purposes.

30 See A. Levin, V. Wieland, and J. Williams (1999).

expectations. In addition, wherever possible the model's behavioral parameters are determined by estimation, which is discussed further in Chapter 4 of McKibbin and Wilcoxen (2002).

9 Appendix II: Calculating Initial Conditions

The following appendix outlines the methods used to calculate the initial conditions for human wealth and effective labour supply. The specification of the age-labour profiles implies that different amounts of effective labour are supplied by adults of different ages. Given the age composition and the age-labour profile of a population, we can determine the aggregate labour supply in an economy. With additional parameters such as the rate of income tax, the interest rate and the mortality rate, we can also calculate the present value of expected lifetime income, thereby deriving starting values for human wealth.

10 Aggregate Labour

As discussed in the text, individual labour supply, and aggregate labour supply, are given by:

(1) $$l(s,t) = e^{ut}[a_1 e^{-\alpha_1(t-s)} + a_2 e^{-\alpha_2(t-s)} + (1-a_1-a_2)e^{-\alpha_3(t-s)}]\ ; \quad (a_i>0,\ \alpha_i>0 \text{ for } i=1 \text{ to } 3)$$

(2) $$L(t) = \int_{-\infty}^{t} l(s,t)n(s,t)\ \ ds$$

It follows that:

(3) $$L_i(t) = \int_{-\infty}^{t} \left[e^{\mu t} a_i e^{-\alpha_i(t-s)}\right] n(s,t)\ \ ds$$

For the purposes of generating initial conditions, we assume that there are no people older than 100 years old, and we discretise the above as:

(4) $$L_i(t) = \sum_{s=t-(100-16)}^{s=t} \left[e^{\mu t} a_i e^{-\alpha_i(t-s)}\right] n(s,t)$$

We use a spreadsheet to calculate how much effective labour an adult would supply, at various ages, and we create a list of individual labour supply values for ages 16 through to 100 for a given year. Using data on the age composition of the relevant countries, we determine the number of people who are of each particular age in a base year. To arrive at a figure for aggregate labour supply, we take a weighted sum of the individual age-specific labour values.

11 Human Wealth

Individual Human Wealth

As discussed in the paper, individual human wealth is given by:

(5) $$h(s,t) = h_1(s,t) + h_2(s,t) + h_3(s,t) + h_4(s,t)$$

where:

(6) $$h_i(s,t) = \int_t^{\infty} e^{-(r+p)(v-t)}[1-\tau(v)]w(v)l_i(s,v)\ dv \qquad [i = 1, 2, 3]$$

and:

(7) $$h_4(s,t) = \int_t^{\infty} e^{-(r+p)(v-t)}[-j(v)]\ dv$$

when lump sum taxes and transfers are zero, as they are in our simulation.

1.1.1. Labour Components of Human Wealth

Starting with the labour components of human wealth (i.e. i=1, 2, 3), we assume that wage and the marginal tax rates are constant over time (or that forward looking agents have no reason, at the initial point in time, to believe that wages or tax rates will change):

(8) $$h_i(s,t) = [1-\tau]w\int_t^{\infty} e^{-(r+p)(v-t)}l_i(s,v)\ dv \qquad [i = 1, 2, 3]$$

(9) $$\begin{aligned} h_i(s,t) &= [1-\tau]w\int_t^{\infty} e^{-(r+p)(v-t)}e^{\mu v}a_i e^{-\alpha_i(v-s)}\ dv \\ &= [1-\tau]a_i w\left[-\frac{1}{r+p+\alpha_i-\mu}e^{-(r+p)(v-t)}e^{\mu v}e^{-\alpha_i(v-s)}\right]_{v=t}^{v=\infty} \end{aligned}$$

So long as $r{+}p{+}\alpha_i{>}\mu$, (which is true of our initial parameter values, as well as most other reasonable initial parameter values), then:

(10) $$\lim_{v\to\infty} \quad e^{-(r+p)(v-t)} e^{\mu v} e^{-\alpha_i (v-s)} = 0$$

This give us:

(11) $$h_i(s,t) = [1-\tau] a_i w \frac{1}{r+p+\alpha_i-\mu} e^{\mu t} e^{-\alpha_i (t-s)}$$

1.1.2. Transfer Component of Human Wealth

When there are no lump sum transfers, no lump sum taxes, and the burden of child support is borne equally across the adult population, the non-labour component of human wealth is given by:

(12) $$h_4(s,t) = \int_t^\infty e^{-(r+p)(v-t)} [-j(v)]\, dv$$

where $j(v)$ represents the amount each adult must pay as a child support transfer each period:

(13) $$\begin{aligned} j(v) &= \frac{c(t)M(t)}{N(t)} \\ &= c(t)\delta(t) \end{aligned}$$

where $c(t)$ represents the amount a child receives, and consumes, each period.

(14) $$\begin{aligned} h_4(s,t) &= -\int_t^\infty e^{-(r+p)(v-t)} c(v)\delta(v)\, dv \\ &= -\int_t^\infty e^{-(r+p)(v-t)} c_o e^{\mu v} \delta(v)\, dv \end{aligned}$$

In steady state, the dependency ratio will be constant. Though the economy may not be in steady state at our starting point, for the purposes of calculating initial conditions, we make the simplifying assumption that adults expect the dependency ratio to remain constant. So:

(15)
$$\begin{aligned} h_4(s,t) &= -c_o\delta\int_t^{\infty} e^{-(r+p)(v-t)}e^{\mu v}\,dv \\ &= -c_o\delta\left[\frac{-1}{r+p-\mu}e^{-(r+p)(v-t)}e^{\mu v}\right]_{v=t}^{v=\infty} \end{aligned}$$

When $r+p>\mu$, (as they are in our simulations), then:

(16)
$$\lim_{v\to\infty} \quad e^{-(r+p)(v-t)}e^{\mu v} = 0$$

Thus:

(17)
$$h_4(s,t) = \frac{-c_o\delta}{r+p-\mu}e^{\mu t}$$

2. Aggregate Human Wealth

Aggregate human wealth is given by:

(18)
$$H(t) = \int_{-\infty}^{t} h(s,t)n(s,t)\,ds$$

which we use in the discrete form:

(19)
$$H(t) = \sum_{s=t-(100-16)}^{s=t} h(s,t)n(s,t)$$

We evaluate this in a manner similar to that which was used for evaluating aggregate labour supply.

Table 1: Selected Dependency Ratios

	2000	2050
	Selected Old Age Dependency Ratios (60+ to 15-59)	
World	16.7%	36.3%
Australia 1	25.9%	52.2%
China	15.5%	55.5%
China, Hong Kong SAR 2	20.7%	69.9%
Dem. Peoples's Rep. of Korea	15.8%	38.6%
France	33.8%	63.7%
Germany	38.0%	77.0%
Greece	38.0%	88.0%
India	12.9%	34.5%
Indonesia	12.4%	38.7%
Japan	37.4%	93.7%
Malaysia	11.1%	35.0%
Philippines	9.7%	32.4%
Republic of Korea	16.1%	65.8%
Russian Federation	29.1%	75.5%
Singapore	15.6%	68.6%
South Africa	9.4%	21.8%
Spain	34.3%	98.9%
Sri Lanka	14.4%	50.1%
Sweden	37.7%	78.1%
Switzerland	34.3%	80.0%
Thailand	12.4%	48.5%
United Kingdom	34.1%	66.5%
United States of America	26.0%	49.2%
Viet Nam	12.6%	41.6%

Source : United Nations Population Division.
World Population Prospects: The 2000 Revision

Table 2: Demographic Data for Japan 1950 to 2050

A. ESTIMATES					Age distribution					
Indicator	Population (thousands) Total	Males	Females	Sex ratio	Percentage aged 0-4	Percentage aged 5-14	Percentage aged 15-24	Percentage 60orover	Percentage 65 or over	Median age (years)
1950	83,625	41,003	42,622	96.2	13.4	22.1	19.6	7.7	4.9	22.3
1955	89,815	44,111	45,704	96.5	10.6	23.0	19.1	8.1	5.3	23.6
1960	94,096	46,176	47,920	96.4	8.5	21.7	18.9	8.9	5.7	25.5
1965	98,881	48,531	50,350	96.4	8.3	17.6	20.2	9.6	6.2	27.3
1970	104,331	51,205	53,126	96.4	8.5	15.5	19.0	10.7	7.1	29.0
1975	111,524	54,880	56,644	96.9	8.9	15.4	15.4	11.7	7.9	30.4
1980	116,807	57,468	59,339	96.8	7.4	16.2	13.8	12.9	9.0	32.6
1985	120,837	59,393	61,444	96.7	6.2	15.4	14.2	14.8	10.3	35.2
1990	123,537	60,658	62,879	96.5	5.3	13.1	15.2	17.4	12.0	37.4
1995	125,472	61,526	63,946	96.2	4.8	11.2	14.7	20.5	14.6	39.7
B. MEDIUM-VARIANT PROJECTIONS										
2000	127,034	62,178	64,856	95.9	4.8	9.9	12.7	23.3	17.2	41.3
2005	127,034	62,178	64,856	95.9	4.6	9.4	11.0	26.3	19.7	42.8
2010	127,998	62,324	65,674	94.9	4.3	9.3	9.9	30.2	22.4	44.5
2015	127,224	61,725	65,499	94.2	4.1	8.9	9.6	32.6	26.0	46.2
2020	125,617	60,693	64,924	93.5	3.8	8.5	9.5	34.0	28.1	48.2
2025	123,444	59,396	64,048	92.7	3.8	8.1	9.3	35.5	29.2	50.2
2030	121,018	58,007	63,011	92.1	4.0	7.9	8.9	37.3	30.4	51.7
2040	115,710	55,163	60,547	91.1	4.2	8.4	8.3	41.6	34.4	53.3
2050	109,722	52,320	57,402	91.1	4.2	8.8	8.9	42.4	36.5	53.2

A. ESTIMATES	Population change per year (thousands)	Births per year (thousands)	Deaths per year (thousands)	Population growth rate (percentage)	Crude birth rate (per 1.000 population)	Crude death rate (per 1.000 population)	Total fertility rate (perwoman)	Net reproduction rate (perwoman)	Infant mortality rate (per 1,000 births)	Life expectancy at birth (years) Both sexes combined
1950-1955	1238	2052	812	1.43	23.7	9.4	2.75	1.19	51.0	63.9
1955-1960	856	1664	715	0.93	18.1	7.8	2.08	0.95	37.0	66.8
1960-1965	957	1662	702	0.99	17.2	7.3	2.02	0.94	25.0	69.0
1965-1970	1090	1805	697	1.07	17.8	6.9	2.00	0.97	16.0	71.1
1970-1975	1439	2073	712	1.33	19.2	6.6	2.07	0.98	12.0	73.3
1975-1980	1057	1733	697	0.93	15.2	6.1	1.81	0.87	9.0	75.5
1980-1985	806	1509	730	0.68	12.7	6.1	1.76	0.85	7.0	76.9
1985-1990	540	1321	773	0.44	10.8	6.3	1.66	0.8	5.0	78.3
199Q-1995	387	1213	863	0.31	9.7	6.9	1.49	0.72	4.0	79.5
B. MEDIUM-VARIANT PROJECTIONS										
1995-2000	312	1213	956	0.25	9.6	7.6	1.39	0.67	3.8	80.5
2000-2005	176	1167	1044	0.14	9.2	8.2	1.32	0.64	3.2	81.6
2005-2010	17	1104	1141	0.01	8.6	8.9	1.32	0.64	3.2	82.6
2010-2015	-155	1036	1244	-0.12	8.1	9.8	1.37	0.66	3.1	82.5
2015-2020	-321	966	1342	-0.25	7.6	10.6	1.42	0.69	3.0	84.3
2020-2025	-435	939	1427	-0.35	7.5	11.5	1.49	0.72	2.9	85.1
2025-2030	-485	958	1497	-0.4	7.8	12.2	1.59	0.77	2.8	85.9
2030-2040	-531	976	1560	-0.45	8.2	13.3	1.71	0.83	2.7	86.9
2040-2050	-598	940	1593	-0.53	8.3	14.2	1.83	0.89	2.5	87.9

Source: Population Division, Department of Economic and Social Affairs of United Nations Secretariat, World Population Prospects: The 2002 Revision.

Table 3: Medium-Variant Projections (2002 UN data)

	Total Pop Size **M+N** (thousands)	Percentage aged 0-4 (%)	Percentage aged 5-14 (%)	Fraction aged 0-14 (Fraction)	Adult Pop Size **N** (thousands)		Adult Pop Growth Rate **n** (rate)	Crude Death Rate (per 1,000)	Crude Death Rate **p** (annualised) (rate)	Adult-Birth Rate **b (residual)** (rate)	Child Death Rate (per 1,000)	Crude child death rate **P (child)** (annualised) (rate)
1950	83,625	13.4	22.1	0.355	53,938							
1955	89,815	10.6	23.0	0.336	59,637	**1950-1955**	2.03%	9.4	0.94%	2.97%	51	0.051
1960	94,096	8.5	21.7	0.302	65,679	**1955-1960**	1.95%	7.8	0.78%	2.73%	37	0.037
1965	98,881	8.3	17.6	0.259	73,271	**1960-1965**	2.21%	7.3	0.73%	2.94%	25	0.025
1970	104,331	8.5	15.5	0.240	79,292	**1965-1970**	1.59%	6.9	0.69%	2.28%	16	0.016
1975	111,524	8.9	15.4	0.243	84,424	**1970-1975**	1.26%	6.6	0.66%	1.92%	12	0.012
1980	116,807	7.4	16.2	0.236	89,241	**1975-1980**	1.12%	6.1	0.61%	1.73%	9	0.009
1985	120,837	6.2	15.4	0.216	94,736	**1980-1985**	1.20%	6.1	0.61%	1.81%	7	0.007
1990	123,537	5.3	13.1	0.184	100,806	**1985-1990**	1.25%	6.3	0.63%	1.88%	5	0.005
1995	125,472	4.8	11.2	0.160	105,396	**1990-1995**	0.89%	6.9	0.69%	1.58%	4	0.004
2000	127,034	4.8	9.9	0.147	108,360	**1995-2000**	0.56%	7.6	0.76%	1.32%	3.8	0.0038
2005	127,914	4.6	9.4	0.140	110,006	**2000-2005**	0.30%	8.2	0.82%	1.12%	3.2	0.0032
2010	128,049	4.3	9.3	0.136	110,634	**2005-2010**	0.11%	8.9	0.89%	1.00%	3.2	0.0032
2015	127,224	4.1	8.9	0.130	110,685	**2010-2015**	0.01%	9.8	0.98%	0.99%	3.1	0.0031
2020	125,617	3.8	8.5	0.123	110,166	**2015-2020**	-0.09%	10.6	1.06%	0.97%	3	0.003
2025	123,444	3.8	8.1	0.119	108,754	**2020-2025**	-0.26%	11.5	1.15%	0.89%	2.9	0.0029
2030	121,017	4.0	7.9	0.119	106,616	**2025-2030**	-0.40%	12.2	1.22%	0.82%	2.8	0.0028
2035	118,445	4.1	8.0	0.121	104,113		-0.46%	12.9	1.29%	0.82%	2.7	0.0027
2040	115,710	4.2	8.4	0.126	101,131	**2030-2040**	-0.53%	13.5	1.35%	0.82%	2.6	0.0026
2045	113,411	4.2	8.7	0.129	98,781		-0.55%	13.9	1.40%	0.84%	2.5	0.0025
2050	109,722	4.2	8.8	0.130	95,458	**2040-2050**	-0.58%	14.4	1.44%	0.86%	2.5	0.0025

Medium-Variant Projections

Source : United Nations Population Division., World Population Prospects: The 2002 Revision

Figure 4 : Implications of a Fertility Decline - Closed Economy versus Open Economy - Theoretical Model

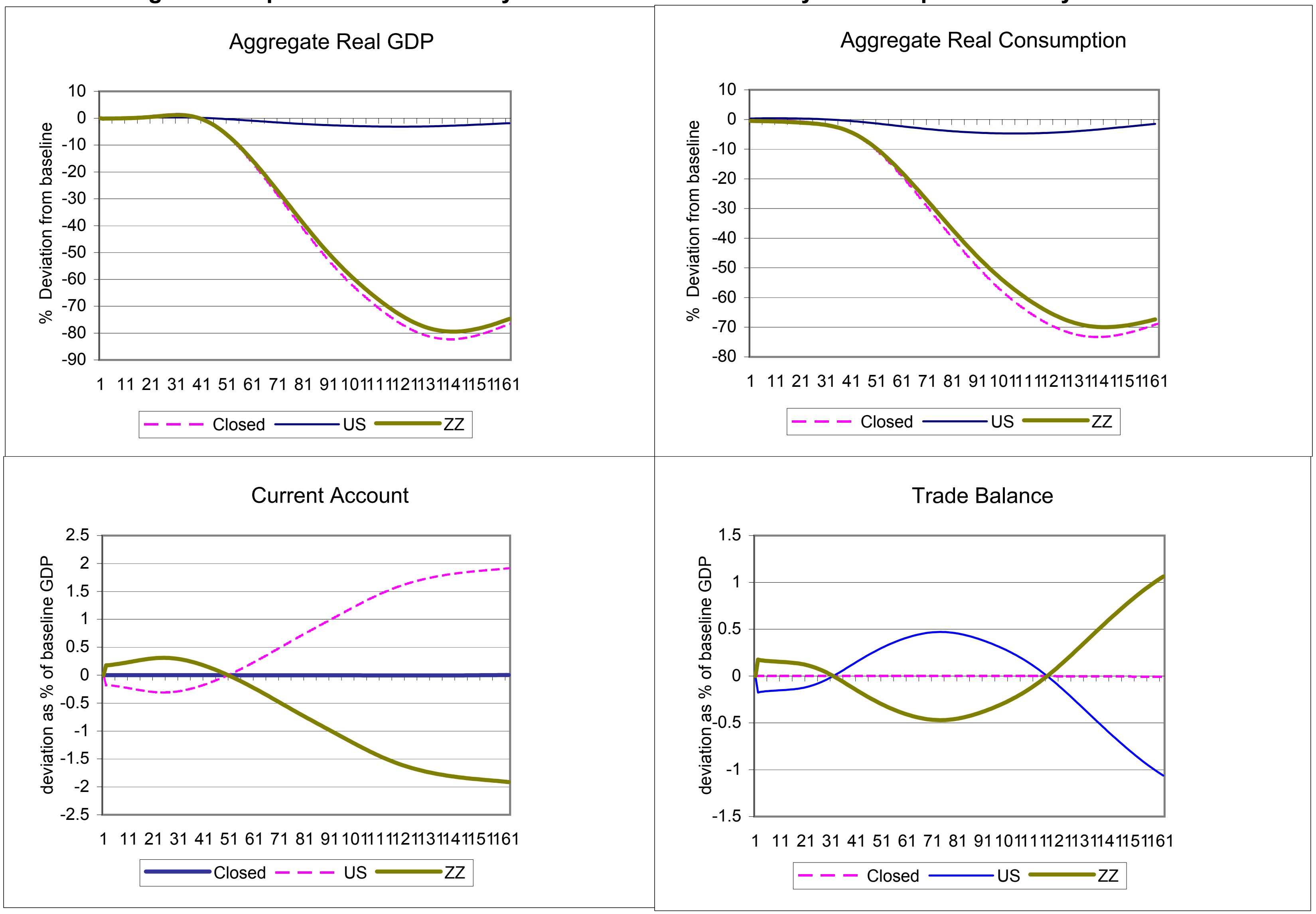


Figure 5 : Implications of a Fertility Decline -Closed Economy versus Open Economy - Theoretical Model

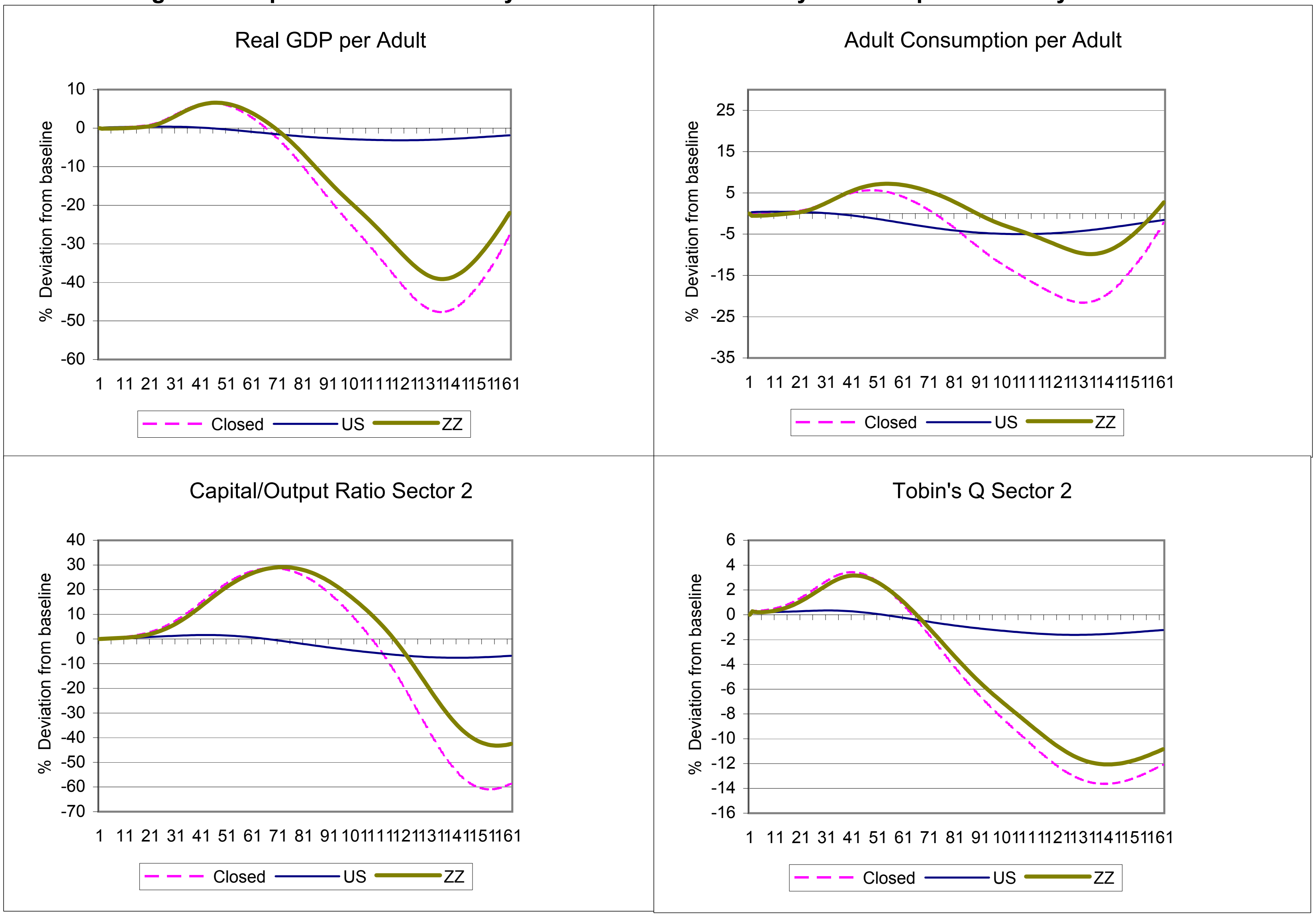


Figure 6: Implications of a Fertility Decline -Closed Economy versus Open Economy - Theoretical Model

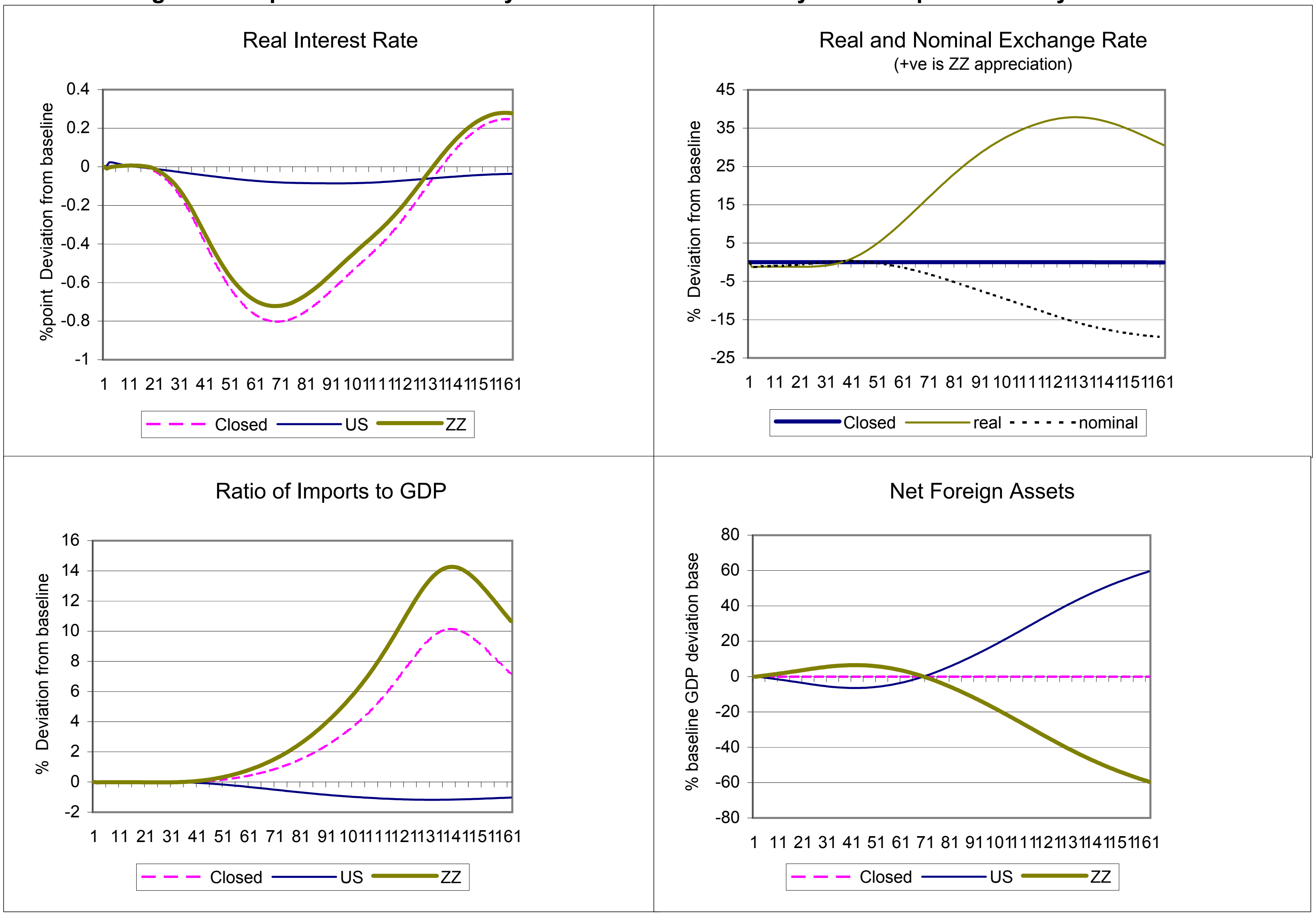


Figure 7 : Implications of a Fertility Decline - Closed Economy versus Open Economy - Symmetric MSG3 Model

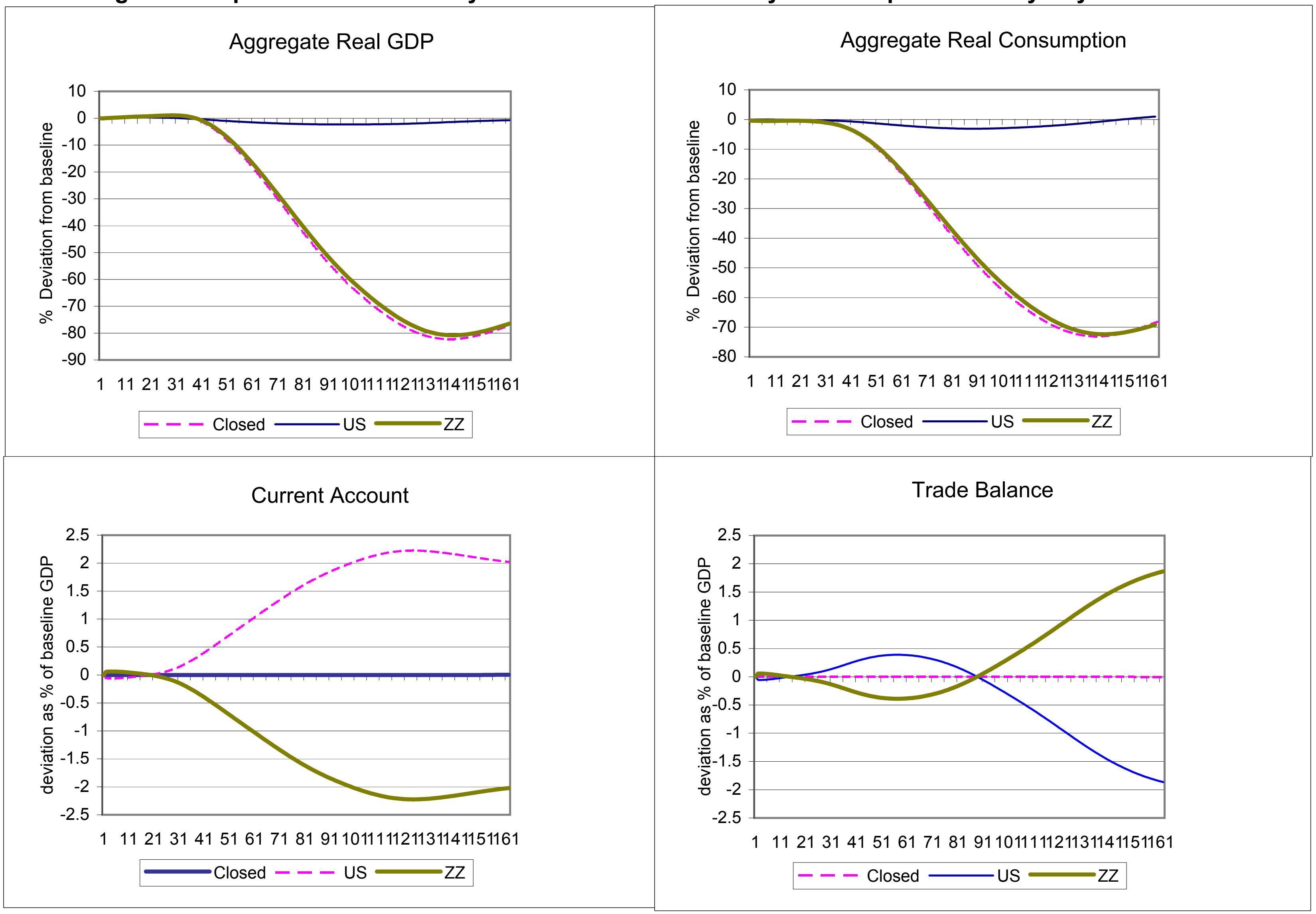


Figure 8 : Implications of a Fertility Decline -Closed Economy versus Open Economy - Symmetric MSG3 Model

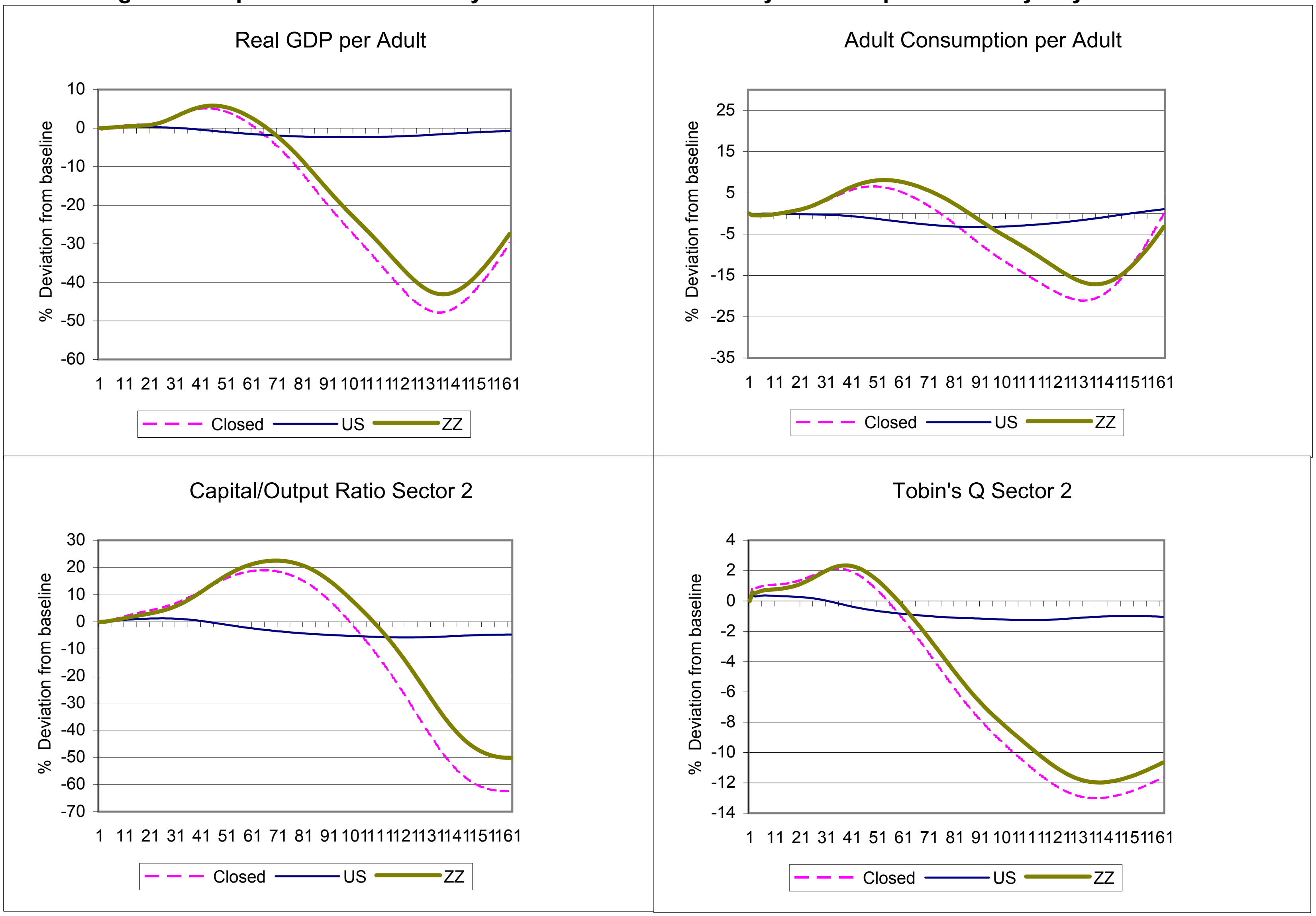


Figure 9: Implications of a Fertility Decline -Closed Economy versus Open Economy - Symmetric MSG3 Model

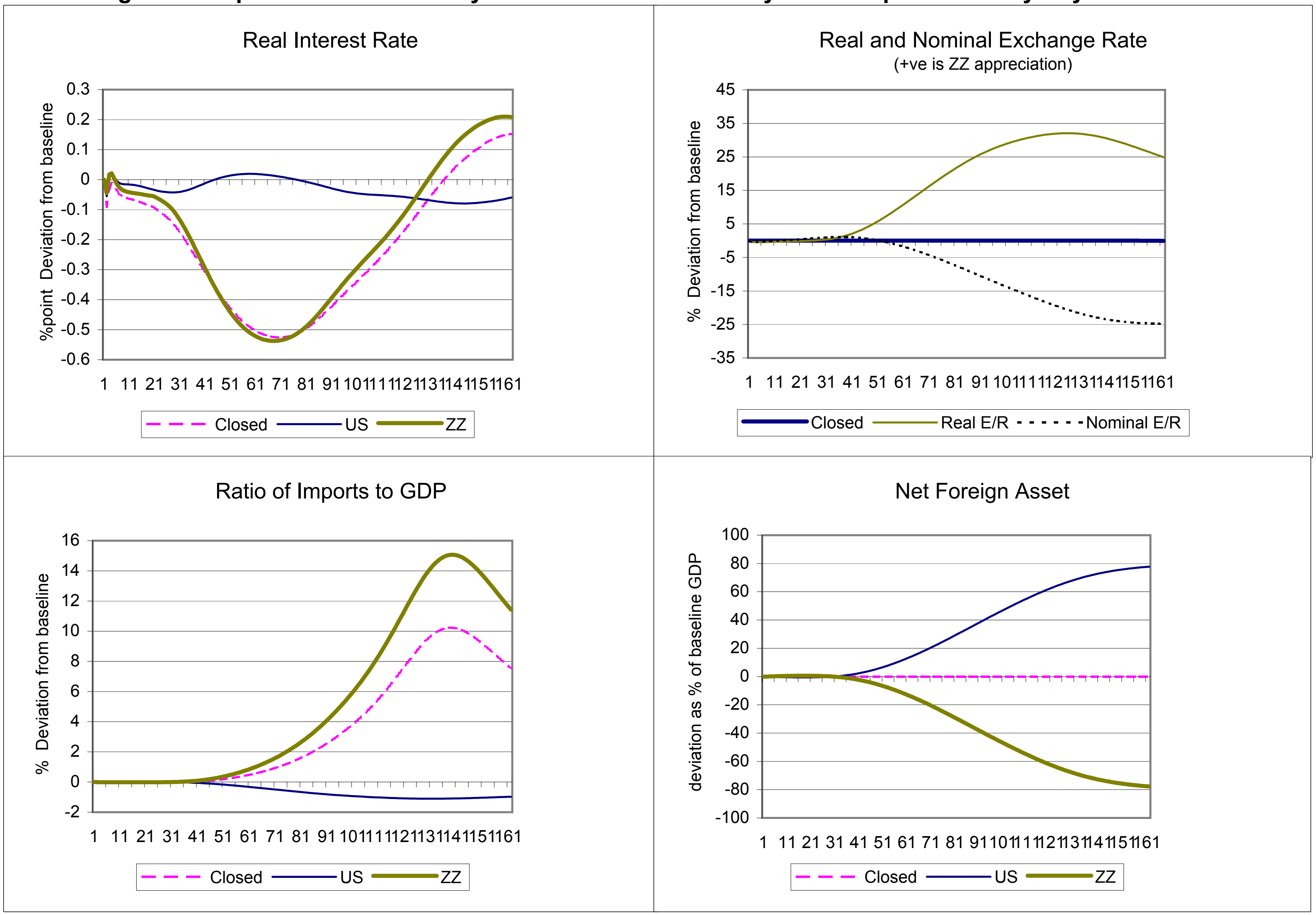


Figure 10 : Implications of a Stylized Fertility Decline -4 Region MSG3 Model

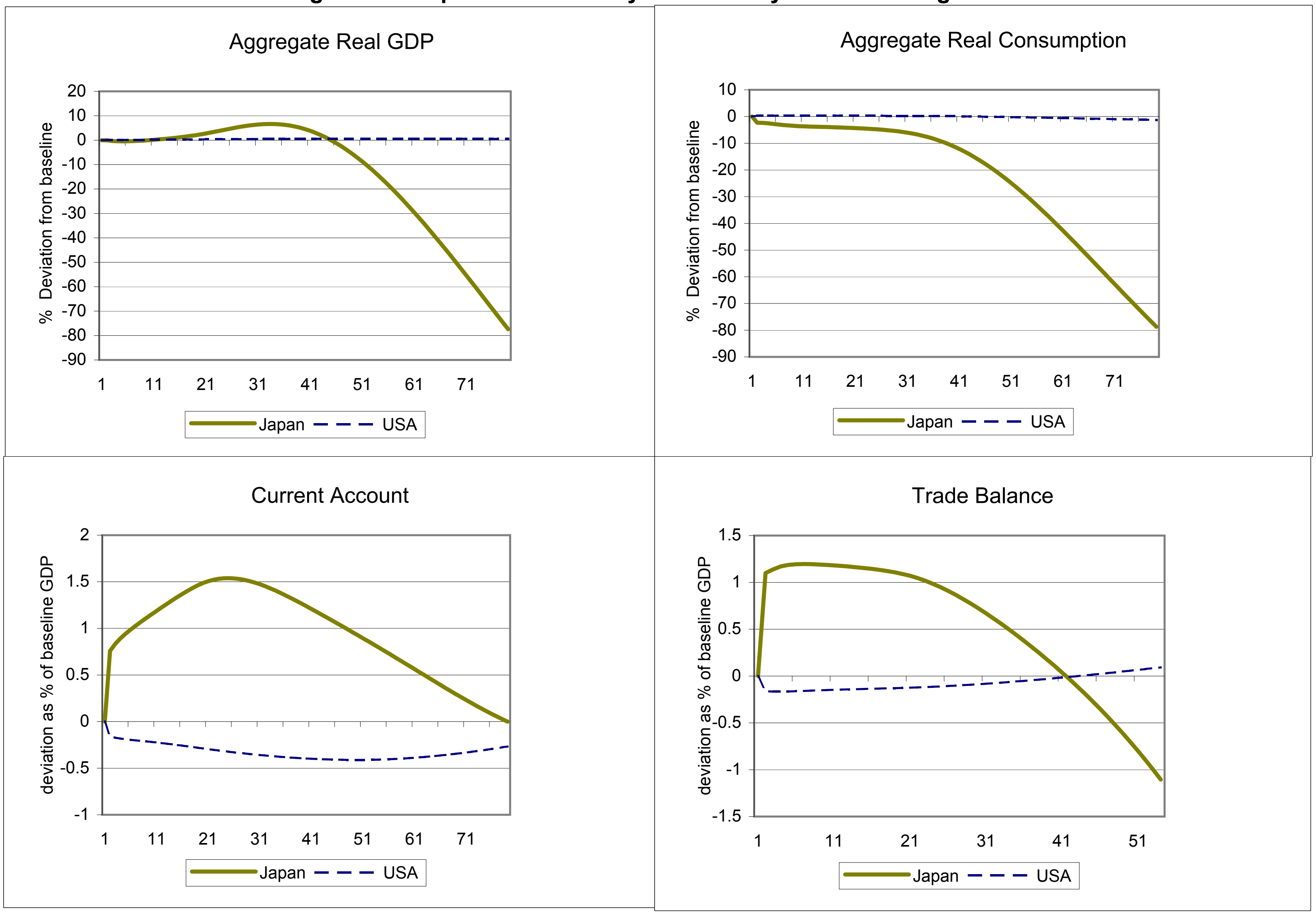


Figure 11 : Implications of a Stylized Fertility Decline -4 Region MSG3 Model

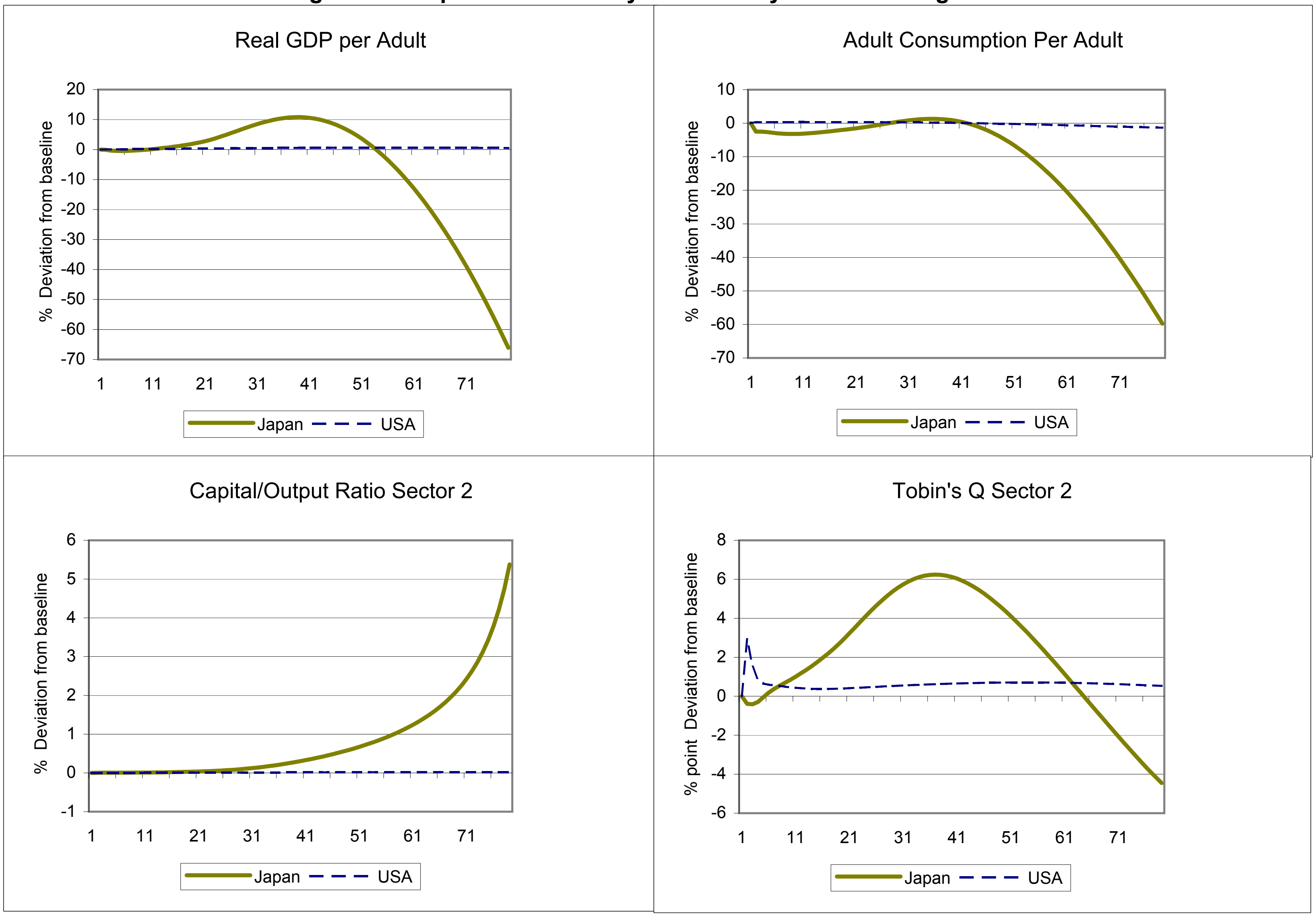


Figure 12 : Implications of a Stylized Fertility Decline - 4 Region MSG3 Model

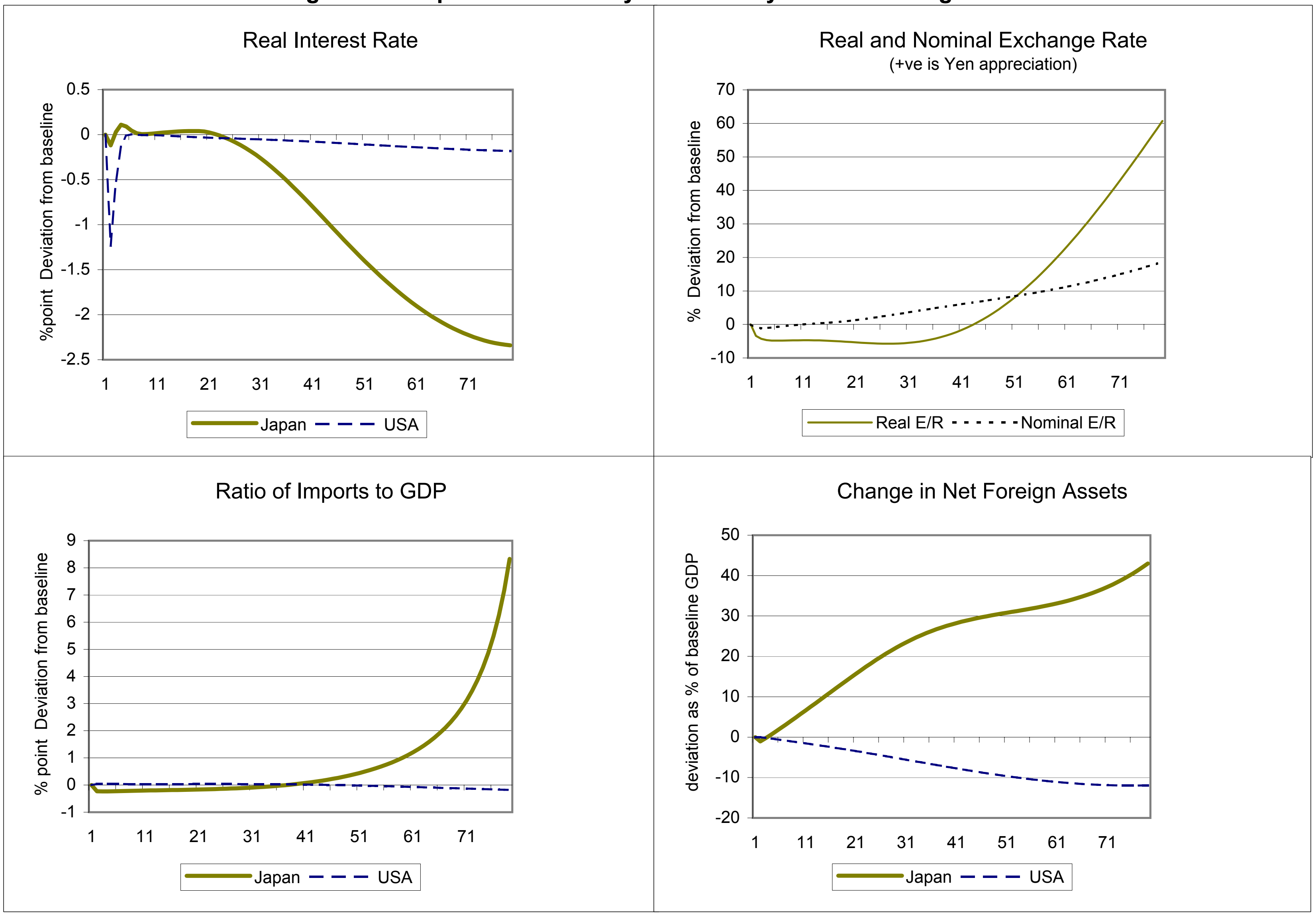


Figure 13
Medium-Variant, Adult Population Projections

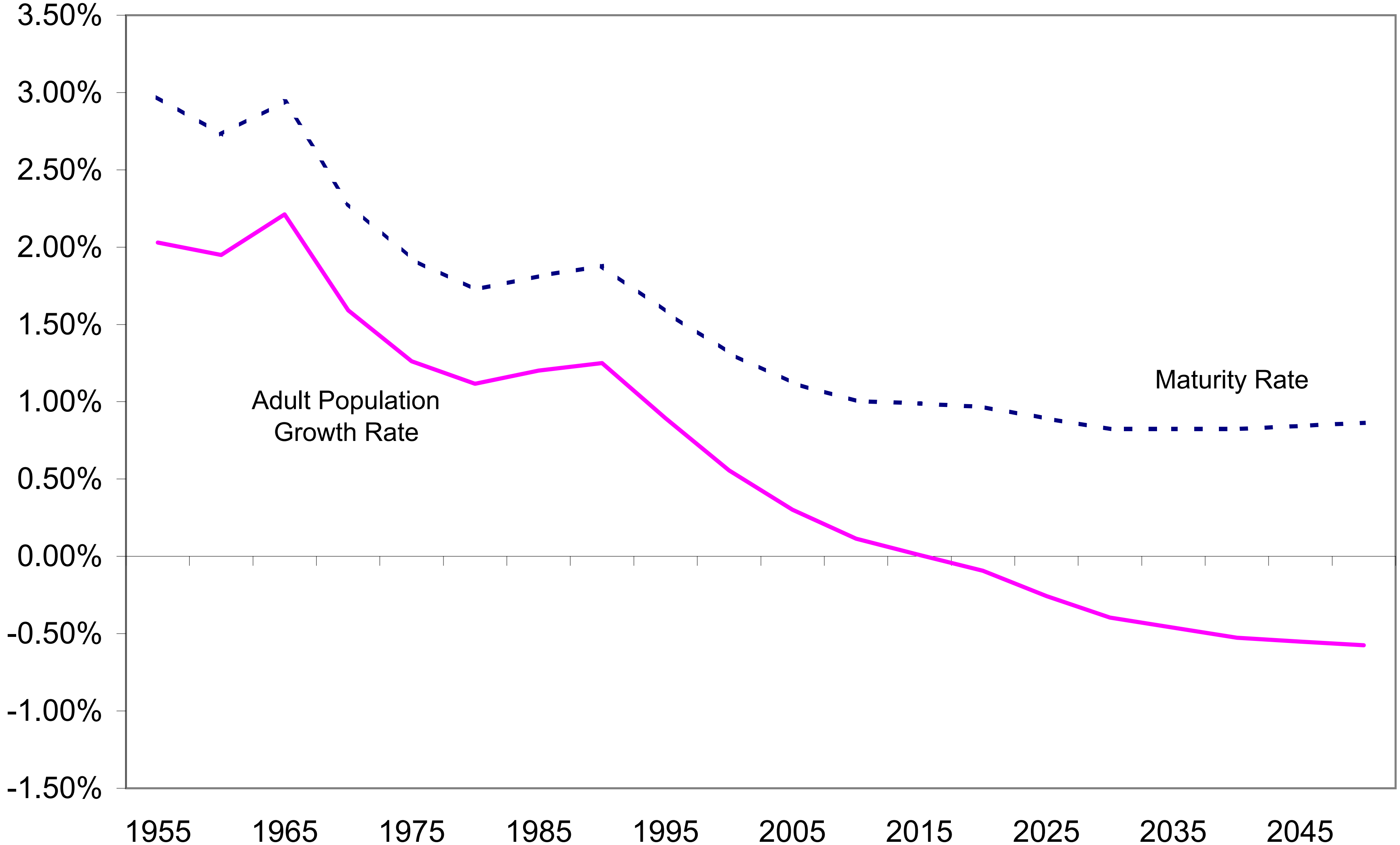
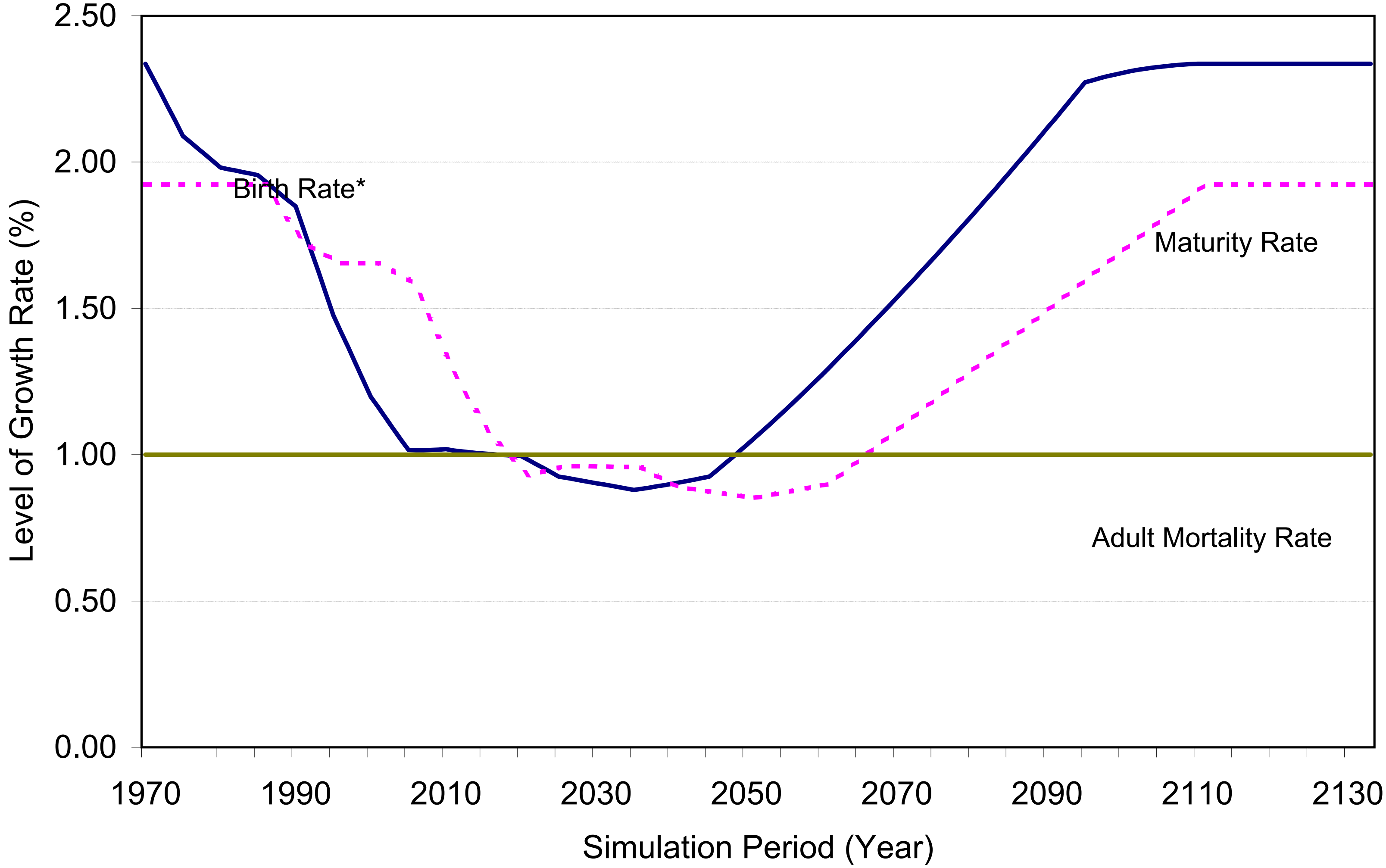


Figure 14
Decline in Japanese Fertility:
Birth Rates, Maturity Rates, Mortality Rates



*Please see text for Birth Rate definition

Figure 15
Effective Labour Supply and Population Profiles for a decline in Japanese Fertility

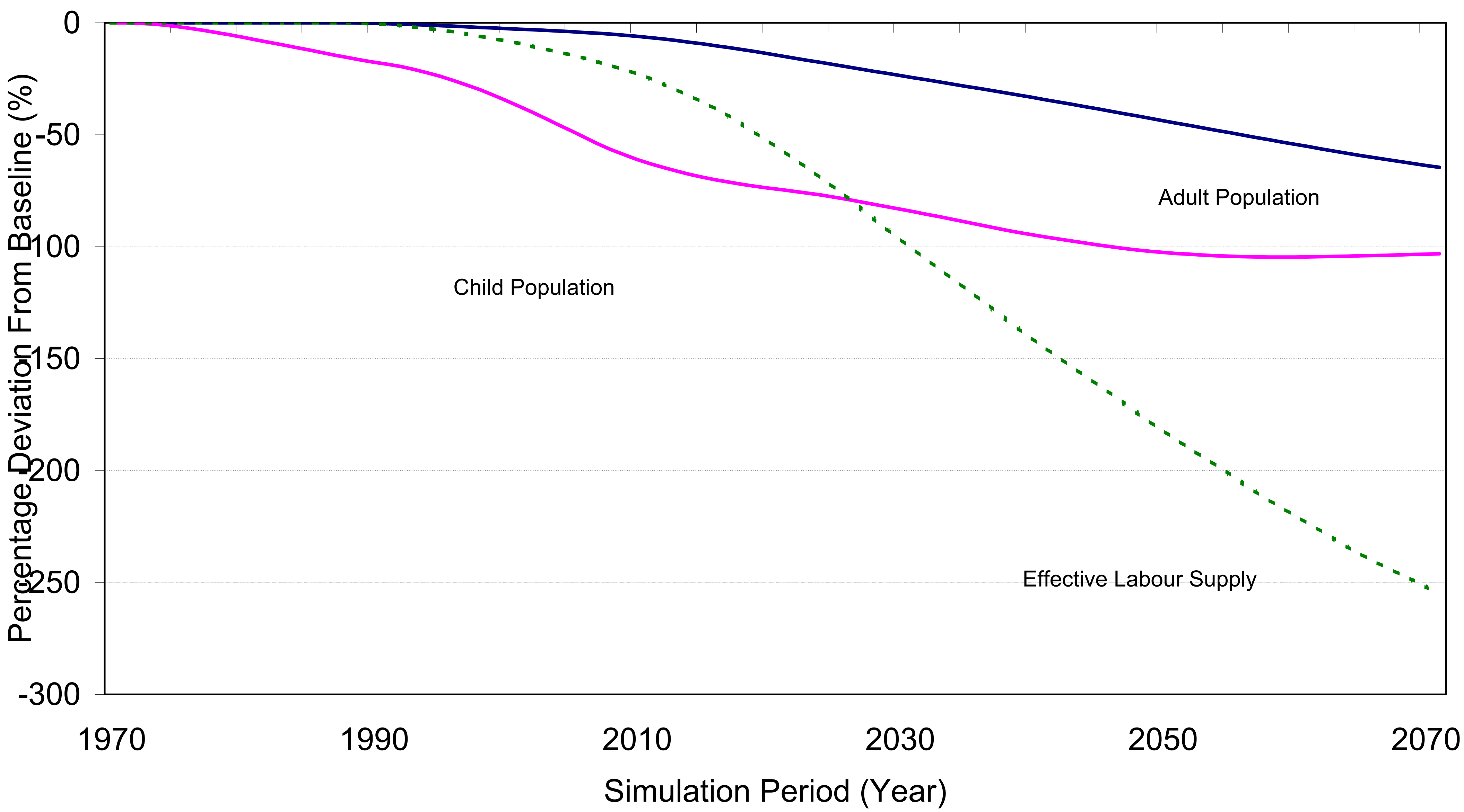


Figure 16 : Contribution of the Actual and Projected Fertility Decline in Japan -MSG3 Model

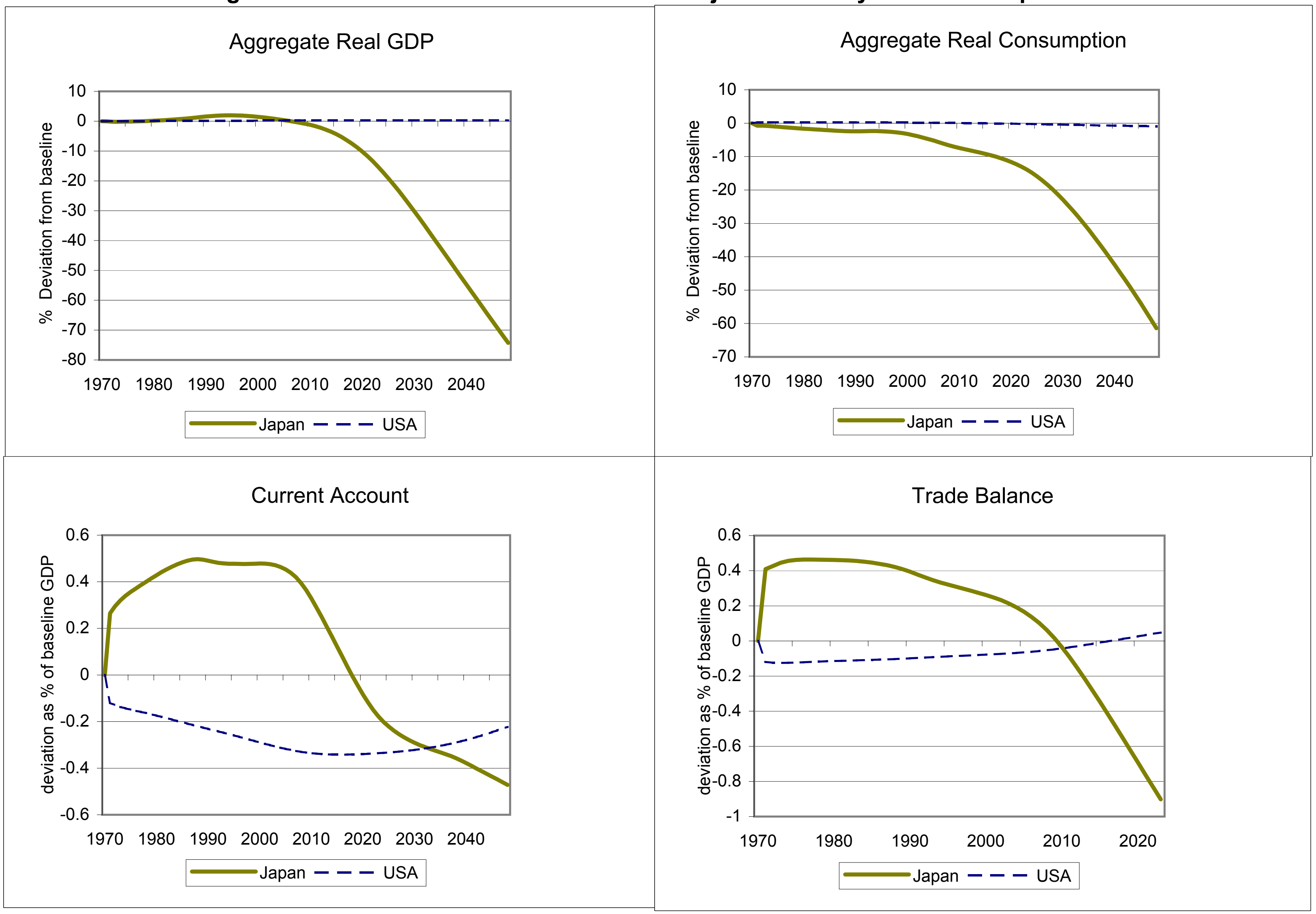


Figure 17 : Contribution of the Actual and Projected Fertility Decline in Japan -MSG3 Model

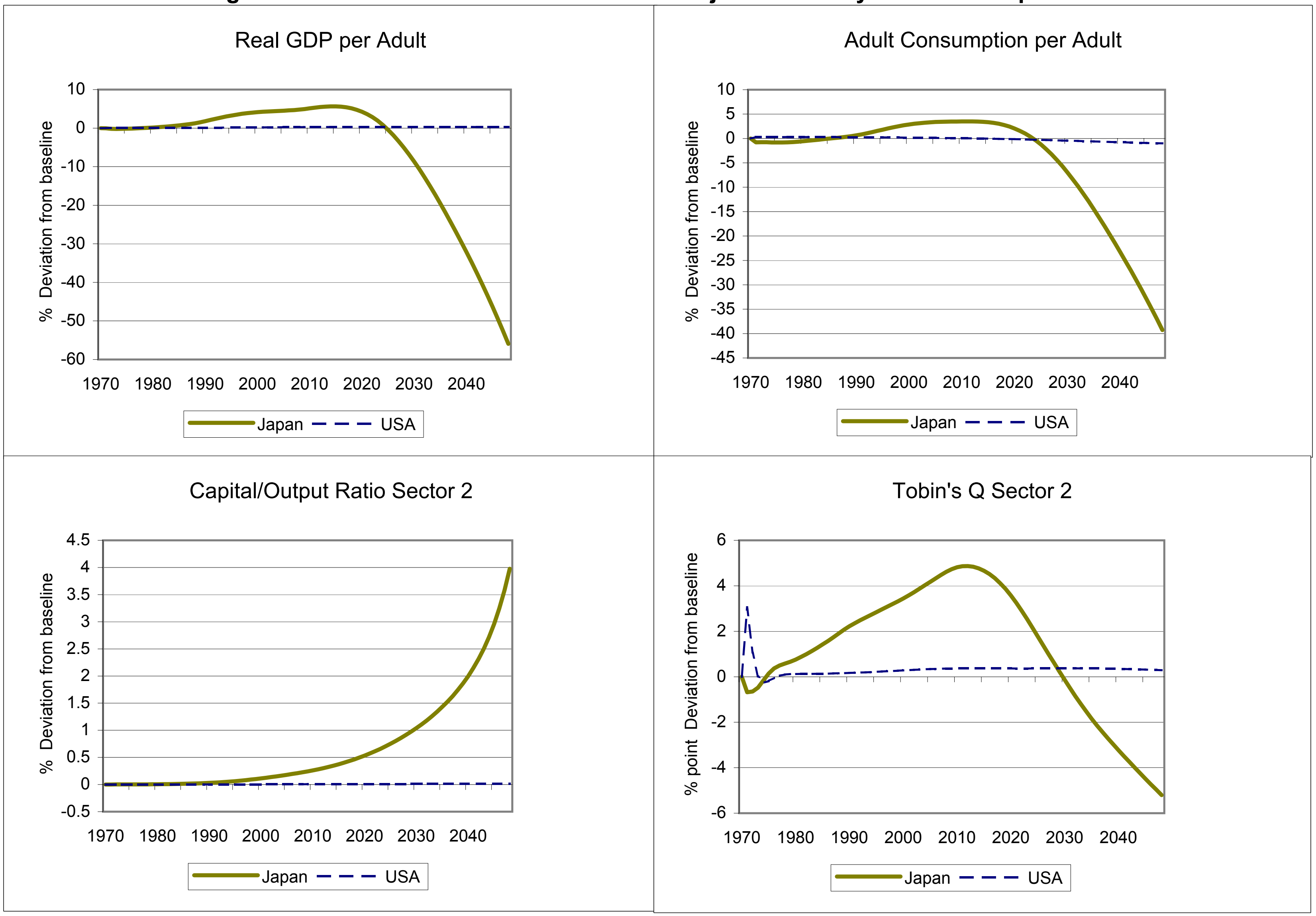


Figure 18 : Contribution of the Actual and Projected Fertility Decline in Japan -MSG3 Model